

# Every Polynomial Ideal is Finitely Generated

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# Scientific Goals and Methods

$$\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right)f(t, x, y) = 1 - \frac{1}{y}t f(t, x, 0)$$

$$\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right) \mathbf{\color{red}f}(t, x, y) = 1 - \frac{1}{y}t \mathbf{\color{red}f}(t, x, 0)$$

$$\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right)f(t, x, y) = 1 - \frac{1}{y}t f(t, x, 0)$$

$$\implies f(t, x, y) = \frac{2x^2yt - x + t + \sqrt{(x-t)^2 - 4t^2x^3}}{2x^2yt\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right)}$$

$$\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right)f(t, x, y) = 1 - \frac{1}{y}t f(t, x, 0)$$

Kernel method

$$\implies f(t, x, y) = \frac{2x^2yt - x + t + \sqrt{(x-t)^2 - 4t^2x^3}}{2x^2yt\left(1 - t\left(\frac{1}{x} + \frac{1}{y} + xy\right)\right)}$$