The Concrete Tetrahedron

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ABSTRACT

We give an overview over computer algebra algorithms for dealing with symbolic sums, recurrence equations, generating functions, and asymptotic estimates, and we will illustrate how to apply these algorithms to problems arising in discrete mathematics.

Categories and Subject Descriptors

I.1.2 [**Computing Methodologies**]: Symbolic and Algebraic Manipulation—*Algorithms*; G.2.1 [**Mathematics of Computing**]: Discrete Mathematics—*Combinatorics*

General Terms

Algorithms

Keywords

Symbolic Sums, Recurrence Equations, Generating Functions, Asymptotic Estimates

1. OVERVIEW

Questions arising in discrete mathematics tend to require calculations involving symbolic sums, recurrence equations, generating functions, and asymptotic estimates. These four mathematical concepts do not stand for their own but rather form the four corners of a compound which we call the concrete tetrahedron. We will survey the most important algorithms which are useful for solving problems in this context: algorithms for obtaining symbolic sums from generating functions, for obtaining recurrence equations from symbolic sums, for obtaining asymptotic estimates from recurrence equations, and so on.

Ideally, the tutorial should cover the four parts of the concrete tetrahedron for polynomial sequences, c-finite sequences, hypergeometric terms, algebraic functions, and for holonomic functions; it should cover the algorithms for univariate sequences as well as their generalizations to the multivariate case; and it should cover algorithmic details as well as real world applications. But this will hardly be possible in the available amount of time. Our plan is to present a representative selection of the material and to give a flavor of the underlying algorithmic principles and the way in which they are put to use.

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Further details on the material covered in the tutorial can be found in the classical textbook [3]. This book focusses more on traditional paper-and-pencil techniques, whereas the recent introductory textbook [4] follows a more algorithmic approach to the subject. Special books on (hypergeometric) summation are [8, 5]. An introduction to the classical theory of generating function is available in [10]. A standard reference on techniques for computing asymptotic estimates is the volume [2]. The relevant original references are available in these books. Unfortunately, we do not have the space to mention them also here.

Concerning software, most general purpose computer algebra systems nowadays include implementations of hypergeometric summation algorithms (Gosper's and Zeilberger's algorithm) as well as facilities for computing various kinds of series expansions. Tools for univariate holonomic sequences and power series are available for Maple in the gfun package [9] and for Mathematica in the a package of Mallinger [7] (since version 7, Mathematica has also builtin tools). The more general algorithms for holonomic and D-finite functions in several variables were implemented by Chyzak [1] in the Mgfun package for Maple and by Koutschan [6] for Mathematica.

2. REFERENCES

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