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# Thinking, Speaking, Writing
# Exercises

## Exercise 1

*Prove*

$$((A \vee B) \to C) \to (A \to C) \wedge (B \to C) \tag{G}$$

*in propositional logic.*

**Proof.** In order to prove the goal (G) we assume the left hand side

$$(A \vee B) \to C \tag{1}$$

and try to deduce the right hand side

$$(A \to C) \wedge (B \to C) \tag{G0}$$

We proceed by case distinction.

- Case 1. We want to prove
  $$A \to C, \tag{G1}$$
  so assuming
  $$A \tag{2}$$
  it remains to show
  $$C. \tag{G2}$$
  To do this, by (1) it is sufficient to show
  $$A \vee B \tag{G3}$$
  for which in turn it is sufficient to prove
  $$A \tag{G4}$$
  which is asserted by (2).
- Case 2. We have to prove
  $$B \to C \tag{G5}$$
  which can be done completely analogous to Case 1 (exchange the roles of $A$ and $B$), so we ommit this part of the prove.

Then we are done. ∎

## Exercise 2

*Prove*

$$(\underset{x}{\forall}\, p[x] \to Q) \to \underset{x}{\exists}(p[x] \to Q) \tag{G}$$

*in first order logic.*

**Proof.** In order to prove (G) we assume the left hand side

$$\underset{x}{\forall}\, p[x] \to Q \tag{1}$$

and try to deduce the right hand side

$$\underset{x}{\exists}(p[x] \to Q). \tag{G1}$$

This means we have to find some $X$ with the property

$$p[X] \to Q. \tag{G2}$$

which in turn means that $X$ should have the property that

$$Q \tag{G3}$$

follows from

$$p[X]. \tag{2}$$

But in order to show $Q$ it is by (1) sufficient to prove

$$\underset{x}{\forall}\, p[x] \tag{G4}$$

instead of $Q$. Now for some arbitrary but fixed $x_0$ it is it remains to show

$$p[x_0] \tag{G5}$$

which is asserted by (2) by using $X = x_0$ there.

This completes the proof. ■