Definite and Indefinite Integration in Mathematica

■ Simplicity

■ Example 1

$$\int \frac{2^x}{\sqrt{1+4^x}}\,dx$$

$$\frac{2^x\,\text{Hypergeometric2F1}\left[\frac{1}{2},\ \frac{\text{Log}[2]}{\text{Log}[4]},\ 1+\frac{\text{Log}[2]}{\text{Log}[4]},\ -4^x\right]}{\text{Log}[2]}$$

Simplify[%]

$$\frac{2^x\,\text{Hypergeometric2F1}\left[\frac{1}{2},\ \frac{\text{Log}[2]}{\text{Log}[4]},\ \frac{\text{Log}[8]}{\text{Log}[4]},\ -4^x\right]}{\text{Log}[2]}$$

Use the fact that $(2^x)^2 = 4^x$

$$\int \frac{2^x}{\sqrt{(2^x)^2+1}}\,dx$$

$$\frac{\text{ArcSinh}[2^x]}{\text{Log}[2]}$$

■ Example 2

$$\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} \frac{1}{2}\sqrt{2-2\cos(2x)}\,dx$$

$$\sqrt{2}$$

■ Example 3

$$\int_{-\infty}^{\infty} \frac{1}{x^4+x^2+x+1}\,dx$$

$$2\,i\left(1728\Big/\left(\left(2\sqrt{\frac{6\left(26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\right)}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+\right.\right.\right.$$

$$\sqrt{\left(\left(6\left(-12\sqrt{6}\left(-43+3\,i\sqrt{771}\right)^{1/3}-26\,2^{1/3}\right.\right.\right.\right.}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)-$$

$$\sqrt{\Bigg(-\Bigg(6\Bigg(-12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}+26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)$$

$$\Bigg(2\sqrt{\frac{6\left(26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\right)}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\sqrt{\Bigg(\Bigg(6\Bigg(-12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}-26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)+$$

$$\sqrt{\Bigg(-\Bigg(6\Bigg(-12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}+26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\,\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)$$

$$\Bigg(\sqrt{\Bigg(\Bigg(6\Bigg(-12\sqrt{6}\,\left(-43+3\,i\,\sqrt{771}\right)^{1/3}-26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\,\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\,\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\right)\Bigg)+$$

$$\sqrt{\Bigg(-\Bigg(\Bigg(6\Bigg(12\sqrt{6}\,\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\,\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\,\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)\Bigg)+$$

$$1728\Bigg/\Bigg(\Bigg(\sqrt{\Bigg(\Bigg(6\Bigg(12\sqrt{6}\,\left(-43+3\,i\,\sqrt{771}\right)^{1/3}-26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\,\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\,\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\,\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\,\sqrt{771}\right)^{1/3}}}\right)\Bigg)+$$

$$\sqrt{\Bigg(-\Bigg(\Bigg(6\Bigg(-12\sqrt{6}\,\left(-43+3\,i\,\sqrt{771}\right)^{1/3}+26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)$$

$$\left(2\sqrt{\frac{6\left(26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\right)}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\right.$$

$$\sqrt{\Bigg(\Bigg(\Bigg(6\Bigg(12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}-26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)-$$

$$\sqrt{\Bigg(-\Bigg(\Bigg(6\Bigg(12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}+26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)$$

$$\left(2\sqrt{\frac{6\left(26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\right)}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\right.$$

$$\sqrt{\Bigg(\Bigg(\Bigg(6\Bigg(12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}-26\,2^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-8\left(-43+3\,i\sqrt{771}\right)^{1/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}-\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)+$$

$$\sqrt{\left(-\left(6\left(12\sqrt{6}\,\left(-43+3\,i\sqrt{771}\right)^{1/3}+26\,2^{1/3}\right.\right.\right.\right.$$

$$\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$8\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}+$$

$$\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-43+3\,i\sqrt{771}\right)^{1/3}\sqrt{\frac{26\,2^{1/3}-4\left(-43+3\,i\sqrt{771}\right)^{1/3}+\left(2\left(-43+3\,i\sqrt{771}\right)\right)^{2/3}}{\left(-43+3\,i\sqrt{771}\right)^{1/3}}}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\pi$$

***N*[%]**

```
1.98668 + 1.74393×10^-16 i
```

$$N\left[\frac{2}{5}\pi\left(\cos\left(\frac{\pi}{5}\right)+1\right)\csc\left(\frac{2\pi}{5}\right)\right]$$

```
2.39027
```

■ Difficult Integrands

■ Example 1

$$\int |x|\,dx$$

```
∫Abs[x] dx
```

$$\int \text{If}[x > 0,\, x,\, -x]\,dx$$

```
∫If[x > 0, x, -x] dx
```

■ Example 2

$$\int \cos(5\,x)\,\mathrm{Ci}(2\,x)\,dx$$

```
1/5 CosIntegral[2 x] Sin[5 x] + 1/10 (-SinIntegral[3 x] - SinIntegral[7 x])
```

■ Example 3

$$\int_{-\infty}^{\infty} \frac{\sin(t)\,e^{2\,i\,t}}{t}\,dt$$

```
0
```

■ Example 4

$$\int \frac{\frac{\partial f(x)}{\partial x}\,g(x) - f(x)\,\frac{\partial g(x)}{\partial x}}{(f(x)+g(x))^2}\,dx$$

```
f[x]
-------
f[x] + g[x]
```

$$\mathrm{Simplify}\left[\frac{\partial\,\%}{\partial x}\right]$$

```
g[x] f'[x] - f[x] g'[x]
-----------------------
        (f[x] + g[x])^2
```

■ Parametric Integrands

■ Example 1

$$\mathrm{Integrate}\left[\frac{\cos(x)}{a^2+x^2}, \{x, -\infty, \infty\}, \mathrm{Assumptions} \to \{a > 0\}\right]$$

```
e^-a π
------
  a
```

Without assumption, *Mathematica* provides even a case distinction:

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{a^2+x^2}\,dx$$

```
If[Arg[a^2] ≠ π, Sqrt[1/a^2] e^-Sqrt[1/a^2] π, ∫_-∞^∞ Cos[x]/(a^2 + x^2) dx]
```

■ Example 2

$$\text{Integrate}\left[x\, e^{2qx-px^2}, \{x, -\infty, \infty\}, \text{Assumptions} \to \{p > 0\}\right]$$

$$\frac{e^{\frac{q^2}{p}} \sqrt{\pi}\, q}{p^{3/2}}$$

Without case distinction, *Mathematica* again finds the restrictions on its own

$$\int_{-\infty}^{\infty} x\, e^{2qx-px^2}\, dx$$

```
If[Re[p] > 0, e^(q^2/p) Sqrt[π] q / p^(3/2), Integrate[e^(2 q x - p x^2) x, {x, -∞, ∞}]]
```

■ Example 3

$$\int \frac{\cot^n(x)}{\sin^2(x)}\, dx$$

$$-\frac{\text{Cot}[x]^{1+n}}{1+n}$$

■ Example 4

$$\text{Integrate}\left[\frac{t^{a-1}}{t+1}, \{t, 0, \infty\}, \text{Assumptions} \to \{a > 0, a < 1\}\right]$$

```
π Csc[a π]
```

Again, appropriate parameter constraints can be found by *Mathematica*.

$$\int_0^{\infty} \frac{t^{a-1}}{t+1}\, dt$$

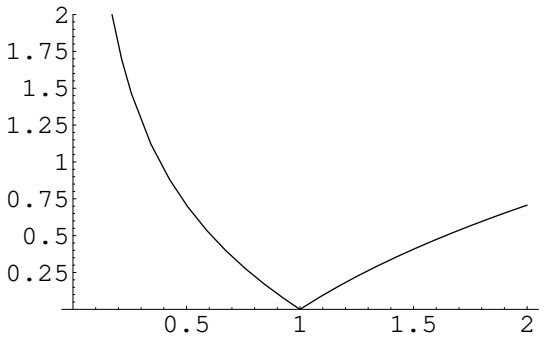
```
If[0 < Re[a] < 1, π Csc[a π], Integrate[t^(-1+a)/(1+t), {t, 0, ∞}]]
```

■ Branch Cut Problems

Consider the real valued function f defined by

$$f(x\_) := \sqrt{x - 2 + \frac{1}{x}}$$

This function has *a* nasty behaviour at $x = 1$:



Mathematica finds an indefinite integral for this function:

$$\int f(x)\,dx$$

$$-\frac{2\,x\,\sqrt{-2+\frac{1}{x}+x}}{-1+x}+\frac{2\,x^2\,\sqrt{-2+\frac{1}{x}+x}}{3\,(-1+x)}$$

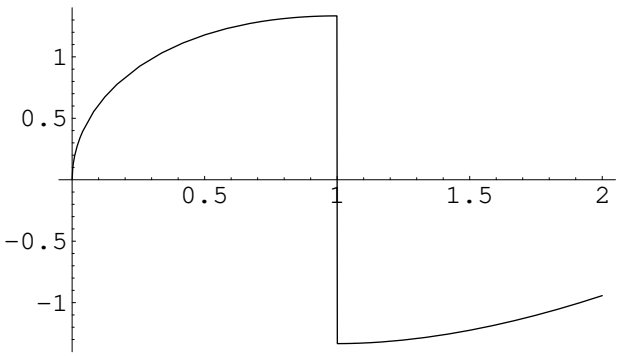
Simplify[%]

$$\frac{2\,(-3+x)\,x\,\sqrt{-2+\frac{1}{x}+x}}{3\,(-1+x)}$$

Define F to be this antiderivative.

$$F(x\_) := \frac{2\,(x-3)\,x\,\sqrt{x-2+\frac{1}{x}}}{3\,(x-1)}$$

This antiderivative has a discontinuity as can be seen from its plot:



However, definite integration works fine and does not struggle over the naive way to apply the fundamental theorem of calculus.

$$\int_0^2 f(x)\,dx$$

$$\frac{8}{3}-\frac{2\sqrt{2}}{3}$$

$$\lim_{x\to 2} F(x)-\lim_{x\to 0} F(x)$$

$$-\frac{2\sqrt{2}}{3}$$