Definite and Indefinite Integration in Maple

Simplicity

Example 1

```
> Int(2^x/sqrt(1+4^x),x);
```

$$\int \frac{2^x}{\sqrt{1+4^x}}\, dx$$

```
> value(%);
```

$$2\,\frac{\displaystyle\sum_{\_k1=0}^{\infty} \frac{\Gamma\left(\_k1+\frac{1}{2}\right)(-1)^{\_k1}\left(1-\mathbf{e}^{\left(-1/2\,x\,(\ln(4)+2\,\_k1\,\ln(4))\left(1-2\,\frac{\ln(2)}{\ln(4)+2\,\_k1\,\ln(4)}\right)\right)}\right)}{\Gamma(\_k1+1)\,(\ln(4)+2\,\_k1\,\ln(4))\left(1-2\,\frac{\ln(2)}{\ln(4)+2\,\_k1\,\ln(4)}\right)}}{\sqrt{\pi}}$$

```
> simplify(%);
```

$$2\,\frac{\displaystyle\sum_{\_k1=0}^{\infty}\left(-\frac{1}{4}\,\frac{(-1+4^{(-\_k1\,x)})\,(-1)^{\_k1}\,\Gamma\left(\_k1+\frac{1}{2}\right)}{\ln(2)\,\_k1\,\Gamma(\_k1+1)}\right)}{\sqrt{\pi}}$$

Use $(2^x)^2 = 4^x$

```
> Int(2^x/sqrt(1+(2^x)^2),x);
```

$$\int \frac{2^x}{\sqrt{1+(2^x)^2}}\, dx$$

```
> value(%);
```

$$\frac{\operatorname{arcsinh}(2^x)}{\ln(2)}$$

Example 2

```
> Int(1/2*sqrt(2-2*cos(2*x)),x=-3/4*Pi..-1/4*Pi);
```

$$\int_{-3/4\,\pi}^{-1/4\,\pi} \frac{1}{2}\sqrt{2-2\cos(2\,x)}\, dx$$

```
> value(%);
```

$$\frac{1}{2}\sqrt{2}\,\sqrt{4}$$

Example 3

```
> Int(1/(1+x+x^2+x^4),x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} \frac{1}{1+x+x^2+x^4}\,dx$$

```
> value(%);
```

$$0$$

This is wrong. The correct answer would have been

$$\frac{2}{5}\pi\left(1+\cos\left(\frac{1}{5}\pi\right)\right)\csc\left(\frac{2}{5}\pi\right)$$

```
> evalf(%);
```

$$2.390265732$$

Difficult Integrands

Example 1

```
> Int(abs(x),x);
```

$$\int |x|\,dx$$

```
> value(%);
```

$$\frac{1}{2}|x|\,x$$

Example 2

```
> Int(cos(5*x)*Ci(2*x),x);
```

$$\int \cos(5\,x)\,\mathrm{Ci}(2\,x)\,dx$$

```
> value(%);
```

$$\int \cos(5\,x)\,\mathrm{Ci}(2\,x)\,dx$$

Example 3

```
> Int(sin(t)/t * exp(2*I*t),t=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} \frac{\sin(t)\,\mathbf{e}^{(2\,I\,t)}}{t}\,dt$$

```
> value(%);
```

$$\int_{-\infty}^{\infty} \frac{\sin(t)\,\mathbf{e}^{(2\,I\,t)}}{t}\,dt$$

Example 4

```
> Int((diff(f(x),x)*g(x)-f(x)*diff(g(x),x))/(f(x)+g(x))^2,x)
;
```

$$\int \frac{\left(\frac{\partial}{\partial x}\,\mathrm{f}(x)\right)\mathrm{g}(x)-\mathrm{f}(x)\left(\frac{\partial}{\partial x}\,\mathrm{g}(x)\right)}{(\mathrm{f}(x)+\mathrm{g}(x))^2}\,dx$$

```
> value(%);
```

$$\int \frac{\left(\frac{\partial}{\partial x}\,\mathrm{f}(x)\right)\mathrm{g}(x)-\mathrm{f}(x)\left(\frac{\partial}{\partial x}\,\mathrm{g}(x)\right)}{(\mathrm{f}(x)+\mathrm{g}(x))^2}\,dx$$

```
> 1/2*(f(x)-g(x))/(f(x)+g(x));
```

$$\frac{1}{2}\,\frac{\mathrm{f}(x)-\mathrm{g}(x)}{\mathrm{f}(x)+\mathrm{g}(x)}$$

```
> simplify(diff(%,x));
```

$$-\frac{-\left(\frac{\partial}{\partial x}\,\mathrm{f}(x)\right)\mathrm{g}(x)+\mathrm{f}(x)\left(\frac{\partial}{\partial x}\,\mathrm{g}(x)\right)}{(\mathrm{f}(x)+\mathrm{g}(x))^2}$$

Parametric Integrands

Example 1

```
> assume(a>0);
> Int(cos(x)/(x^2+a^2),x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2+a\sim^2}\,dx$$

```
> value(%);
```

$$\frac{\pi\,(-\sinh(a\sim)+\cosh(a\sim))}{a\sim}$$

```
> convert(%, exp);
```

$$\frac{\pi}{\mathbf{e}^{a\sim}\,a\sim}$$

```
> a := 'a':
```

Example 2

```
> assume(p>0);
> Int(x*exp(-p*x^2+2*q*x),x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} x\, \mathbf{e}^{(-p\sim x^2 + 2\, q\, x)}\, dx$$

```
> value(%);
```

$$\frac{\mathbf{e}^{\left(\frac{q^2}{p\sim}\right)} q \sqrt{\pi}}{p\sim^{(3/2)}}$$

```
> p := 'p':
```

Example 3

```
> assume(a>0, a<1);
> Int(t^(a-1)/(1+t),t=0..infinity);
```

$$\int_{0}^{\infty} \frac{t^{(-1 + a\sim)}}{1 + t}\, dt$$

```
> value(%);
```

$$\int_{0}^{\infty} \frac{t^{(-1 + a\sim)}}{1 + t}\, dt$$

```
> a := 'a':
```

Example 4

```
> assume(n, integer);
> Int(cot(x)^n/sin(x)^2,x);
```

$$\int \frac{\cot(x)^{n\sim}}{\sin(x)^2}\, dx$$

```
> value(%);
```

$$\int \frac{\cot(x)^{n\sim}}{\sin(x)^2}\, dx$$

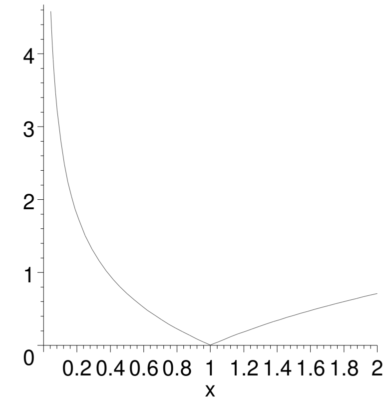
```
> n := 'n':
```

Branch Cut Problems

```
> f := x-> sqrt(x+1/x-2);
```

$$f := x \to \sqrt{x + \frac{1}{x} - 2}$$

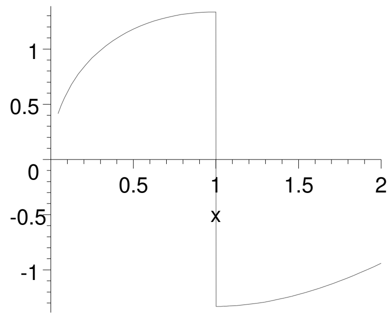
```
> plot(f(x),x=0..2);
```




```
> Int(f(x),x) = int(f(x),x);
```

$$\int \sqrt{x+\frac{1}{x}-2}\,dx=\frac{2}{3}\,\frac{\sqrt{\frac{x^2+1-2\,x}{x}}\,x\,(x-3)}{x-1}$$

```
> F := x -> 2/3*sqrt((x^2+1-2*x)/x)*x*(x-3)/(x-1):
> plot(F(x), x=0..2);
```




```
> Int(f(x),x=0..2);
```

$$\int_0^2 \sqrt{x+\frac{1}{x}-2}\,dx$$

```
> value(%);
```

$$-\frac{2}{3}\sqrt{2}+\frac{8}{3}$$

```
> Limit('F(x)',x=2, left) - Limit('F(x)', x=0, right);
```

$$\left(\lim_{x\to 2-}\mathrm{F}(x)\right)-\left(\lim_{x\to 0+}\mathrm{F}(x)\right)$$

```
> value(%);
```

$$-\frac{2}{3}\sqrt{2}$$