Manuel Kauers
RISC-Linz

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Algorithmic Algebraic Geometry
Solution 4

The following lines are exactly the bitangents to the plane algebraic curve

$$2x^4 + 2y^4 + 2 - 3x^2y^2 - 3x^2 - 3y^2 = 0.$$

Note that in this case all real lines are expressible over $\mathbb{Q}(\sqrt{2})$.

$$t \mapsto \binom{0}{0} + t\binom{-1+\sqrt{2}}{1} \qquad t \mapsto \binom{0}{0} + t\binom{-1-\sqrt{2}}{1},$$

$$t \mapsto \binom{0}{1/2\sqrt{2}} + t\binom{1}{-1}, \qquad t \mapsto \binom{0}{-1/2\sqrt{2}} + t\binom{1}{-1},$$

$$t \mapsto \binom{0}{1} + t\binom{1}{-1/2\sqrt{2}}, \qquad t \mapsto \binom{0}{1} + t\binom{1}{1/2\sqrt{2}},$$

$$t \mapsto \binom{0}{-1} + t\binom{1}{-1}, \qquad t \mapsto \binom{0}{-1+\sqrt{2}} + t\binom{1}{0},$$

$$t \mapsto \binom{0}{-1-\sqrt{2}} + t\binom{1}{0}, \qquad t \mapsto \binom{0}{-1} + t\binom{1}{-1/2\sqrt{2}},$$

$$t \mapsto \binom{0}{-1} + t\binom{1}{1/2\sqrt{2}}, \qquad t \mapsto \binom{0}{-\sqrt{2}} + t\binom{1/2\sqrt{2}}{1},$$

$$t \mapsto \binom{0}{\sqrt{2}} + t\binom{-1/2\sqrt{2}}{1}, \qquad t \mapsto \binom{0}{1} + t\binom{1}{-1},$$

$$t \mapsto \binom{0}{-\sqrt{2}} + t\binom{-1/2\sqrt{2}}{1}, \qquad t \mapsto \binom{0}{\sqrt{2}} + t\binom{1/2\sqrt{2}}{1},$$

$$t \mapsto \binom{0}{0} + t\binom{1-\sqrt{2}}{1}, \qquad t \mapsto \binom{0}{0} + t\binom{1+\sqrt{2}}{1},$$

$$t \mapsto \binom{0}{1/2\sqrt{2}} + t\binom{1}{1}, \qquad t \mapsto \binom{0}{-1/2\sqrt{2}} + t\binom{1}{1},$$

$$t \mapsto \binom{0}{-1} + t\binom{1}{1}, \qquad t \mapsto \binom{0}{1-\sqrt{2}} + t\binom{1}{0},$$

$$t \mapsto \binom{0}{1+\sqrt{2}} + t\binom{1}{0}, \qquad t \mapsto \binom{-1+\sqrt{2}}{0} + t\binom{0}{1},$$

$$t \mapsto \binom{-1-\sqrt{2}}{0} + t\binom{0}{1}, \qquad t \mapsto \binom{1-\sqrt{2}}{0} + t\binom{0}{1},$$

$$t \mapsto \binom{1+\sqrt{2}}{0} + t\binom{0}{1}, \qquad t \mapsto \binom{0}{1} + t\binom{1}{1}.$$