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Algorithmic Combinatorics
Homework 8

Exercise 1 Let X be a G-set and G_Y be the set stabilizer of $Y \subseteq X$. Prove $G_Y \leq G$.

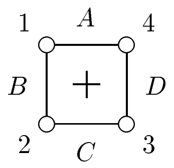
G_Y is clearly a semigroup. By $1_G(x) = x$ $(x \in X \supseteq Y)$ we have $1_G \in G_Y$, so G_Y is a monoid. Finally, if $g \in G_Y$, then all $y \in Y$ satisfy $gy = y$, so $y = g^{-1}y$ for all $y \in Y$, so $g^{-1} \in G_Y$. Therefore, G_Y is a group.

Exercise 2 Let G be a finite group and $H \leq G$. Show that $|G/H| = |G|/|H|$.

G/H is a partition, $[1_G] = H$, and every equivalence class has the same number of elements. For the last claim, observe that $x \mapsto gx$ is a bijection between H and gH.

Exercise 3 What is the symmetry group H of the corners of a square? What is the symmetry group H' of the edges of a square?

Consider the following square:



We find the following symmetry groups by inspection:

$$\begin{aligned} H &= \{\mathrm{id}, (1\,2)(3\,4), (1\,4)\,(2\,3), (1\,3)(2\,4), (1\,2\,3\,4), (1\,4\,3\,2)\} \\ H' &= \{\mathrm{id}, (A\,C), (B\,D), (A\,D)(B\,C), (A\,B\,C\,D), (A\,C)(B\,D), (A\,D\,C\,B)\}. \end{aligned}$$

Observe that H and H' cannot be isomorphic, because they do not have the same number of elements.