Guessing with little data

Manuel Kauers · Institute for Algebra · JKU

Joint work with Christoph Koutschan

$$a_0 = 1,\ (n+2)a_{n+1} - (4n+2)a_n = 0$$

$$\Downarrow$$

n	0	1	2	3	4	5	6	7
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Instantiate the ansatz

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for $n = 0, \ldots, 6$ to get a linear system for the four undetermined coefficients $c_{00}, c_{01}, c_{10}, c_{11}$.

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$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 2 \\ 2 & 4 & 5 & 10 \\ 5 & 15 & 14 & 42 \\ 14 & 56 & 42 & 168 \\ 42 & 210 & 132 & 660 \\ 132 & 792 & 429 & 2574 \end{pmatrix} \begin{pmatrix} c_{00} \\ c_{01} \\ c_{10} \\ c_{11} \end{pmatrix} = 0$$

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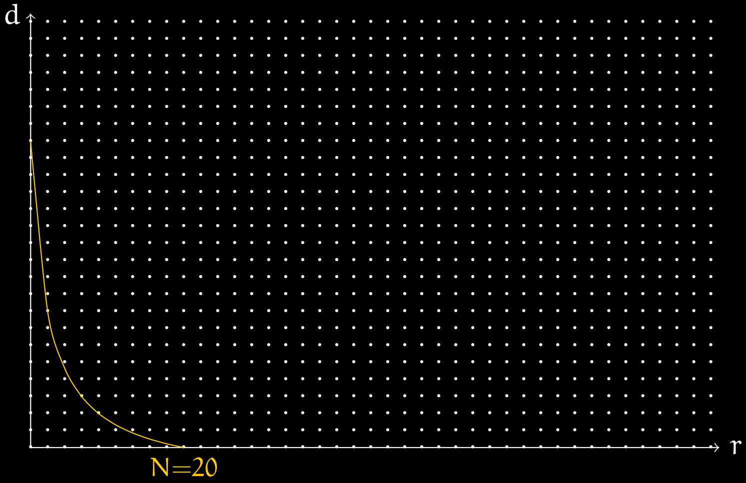
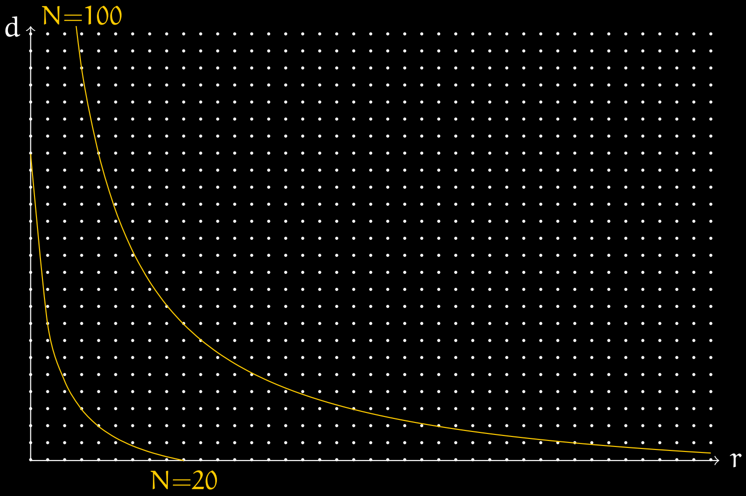
The linear system is overdetermined iff $(r+1)(d+2) \leq N+1$.


d
r


d
r
N=20



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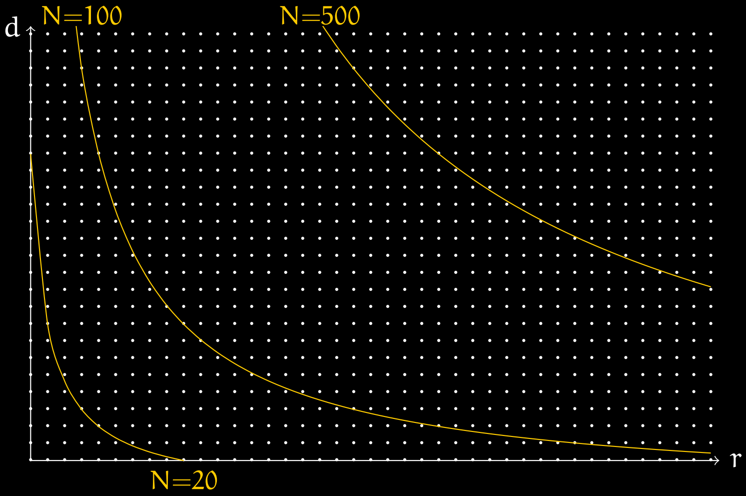
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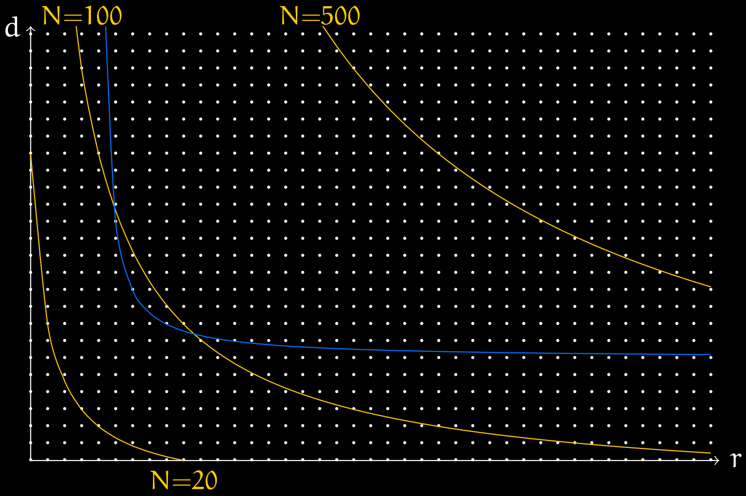

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But the last one is accessible via LLL :-)

Example

The numbers $a_n = \sum_k \binom{n}{k}^3$ satisfy a recurrence of order 2 and degree 2.

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Suppose we only know $a_0, \ldots, a_6$.

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 0 & 0 & 10 & 0 & 0 \\ 2 & 2 & 2 & 10 & 10 & 10 & 56 & 56 & 56 \\ 10 & 20 & 40 & 56 & 112 & 224 & 346 & 692 & 1384 \\ 56 & 168 & 504 & 346 & 1038 & 3114 & 2252 & 6756 & 20268 \\ 346 & 1384 & 5536 & 2252 & 9008 & 36032 & 15184 & 60736 & 242944 \end{pmatrix} \begin{pmatrix} c_{00} \\ c_{01} \\ c_{02} \\ c_{10} \\ c_{11} \\ c_{12} \\ c_{20} \\ c_{21} \\ c_{22} \end{pmatrix} = 0$$

Example

A basis for the $\mathbb{Z}$-module of all solutions in $\mathbb{Z}^9$ is

$$\begin{pmatrix} 2 \\ 0 \\ 0 \\ 67069 \\ -52693 \\ -45994 \\ -13414 \\ 13424 \\ 5636 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 231310 \\ -181747 \\ -158629 \\ -46262 \\ 46300 \\ 19438 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \\ 232560 \\ -182728 \\ -159486 \\ -46512 \\ 46550 \\ 19543 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 434140 \\ -341119 \\ -297729 \\ -86828 \\ 86900 \\ 36483 \end{pmatrix}$$

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A basis for the $\mathbb{Z}$-module of all solutions in $\mathbb{Z}^9$ is

$$\begin{pmatrix} -8 \\ -16 \\ -8 \\ -16 \\ -21 \\ -7 \\ 4 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} -8 \\ 21 \\ -11 \\ -6 \\ -29 \\ 1 \\ 2 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 12 \\ 12 \\ -34 \\ -46 \\ 27 \\ 21 \\ 8 \\ -6 \\ -2 \end{pmatrix}, \begin{pmatrix} -42 \\ 14 \\ 58 \\ -29 \\ -17 \\ 40 \\ 10 \\ -4 \\ -6 \end{pmatrix}$$

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Indeed,

$$(-8-16n-8n^2)a_n+(-16-21n-7n^2)a_{n+1}+(4+4n+n^2)a_{n+2}=0$$

is a correct recurrence.

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There is a theoretical and a pragmatic answer.

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The linear system must be such that the "true" coefficient vector is significantly shorter than a "generic" solution.

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Theorem (Siegel's lemma). For every $A \in \mathbb{Z}^{n \times m}$ with $m > n$ there exists $x \in \ker_{\mathbb{Z}} A \setminus \{0\}$ with $\|x\|_\infty \leq (m \, \|A\|_\infty)^{m/(m-n)}$.

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Then the corresponding linear system has a nonzero solution x with

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For fixed r and N, this bound converges to 1 for $d \to \infty$.

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Interpretation. Unless the length of the coefficients of the "true" recurrence is significantly shorter than the bound, we cannot expect to discover it.

- The convergence to 1 for $d \to \infty$ implies a restriction on the range of degrees that can be reasonably testet.
- The difference between the length of the shortest vector found and the bound can be used as a confidence measure.

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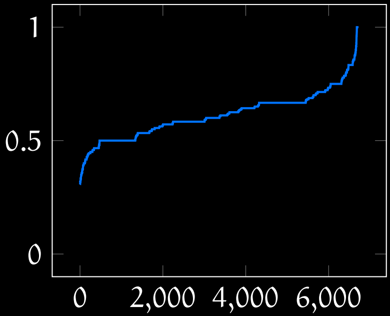
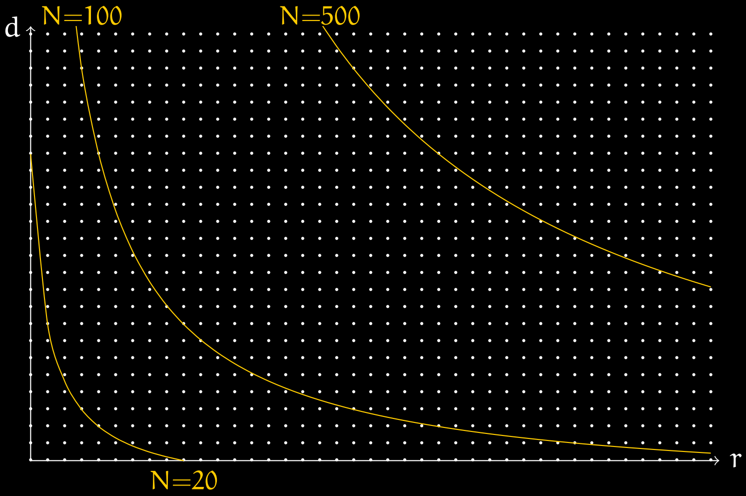
Outcome:

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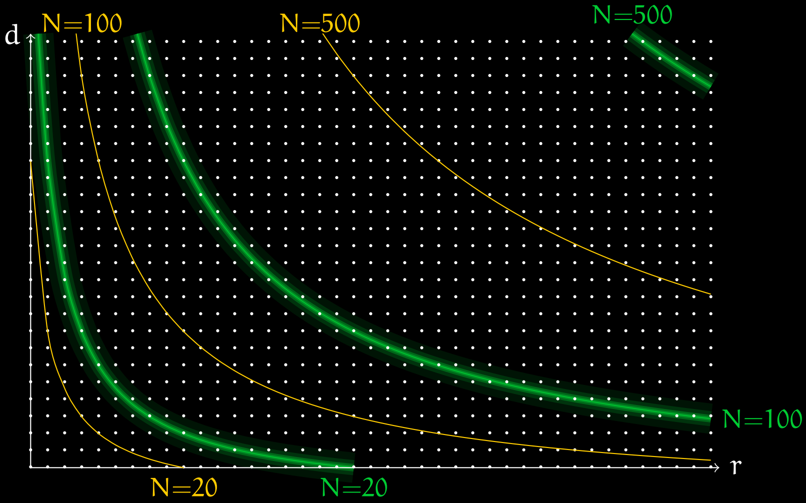

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LLL-based guessing doesn't succeed either.

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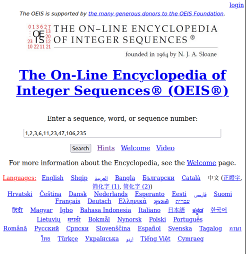

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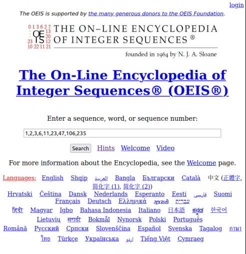

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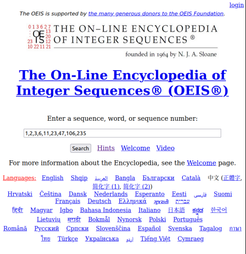

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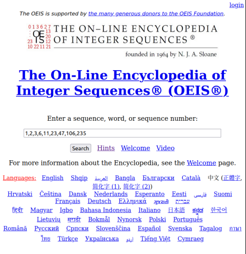

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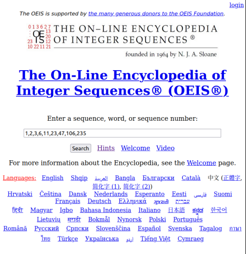

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500 of them have between 30 and 90 known terms.

We applied our LLL-based guesser to these.

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This was the case for about 20 sequences.

18 of these cases look suspicious in one way or another, and we don't think they are correct.

In the other two cases, we think we found something interesting.

A307717

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Only $a_1, \ldots, a_{70}$ are known:

4,	5,	37,	65,	272,	300,	957,
0,	0,	0,	0,	0,	0,	0,
2,	13,	27,	138,	177,	613,	577,
0,	0,	0,	0,	0,	0,	0,
5,	9,	60,	94,	365,	379,	1170,
0,	0,	0,	0,	0,	0,	0,
3,	22,	43,	197,	233,	772,	698,
0,	0,	0,	0,	0,	0,	0,
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Example: $a_{15} = 9$ because:

$$\begin{aligned}
10000001^2 &= 100000020000001 \\
10011001^2 &= 100220141022001 \\
10100101^2 &= 102012040210201 \\
10111101^2 &= 102234363432201 \\
11000011^2 &= 121000242000121 \\
11011011^2 &= 121242363242121 \\
11100111^2 &= 123212464212321 \\
11111111^2 &= 123456787654321 \\
20000002^2 &= 400000080000004
\end{aligned}$$

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Conjecture:

$$a_n = \begin{cases} 0 & \text{if } n = 0 \bmod 2 \\ \frac{1}{192}(195 + 203n - 15n^2 + n^3) & \text{if } n = 1 \bmod 4 \\ \frac{1}{384}(501 + 107n - 9n^2 + n^3) & \text{if } n = 3 \bmod 4 \end{cases}$$

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A189281

$$a_n := \#\{\, \pi \in S_n : \pi(k+2) - \pi(k) \neq 2 \text{ for all } k \,\}$$

A189281

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75,
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2729,
20906,
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2847624899,
39282739034,
581701775369,
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154873904848803,
2762800622799362,
52071171437696453,
1033855049655584786,
21567640717569135515,
471630531427793184474,
10787660036599729160073,
257590656485400508526570,
6409633590481106885238443,
165928838963556686281573922,
4462073606461933066205164757,
124470791290376112779747519538,
3597058248632667485834774744787,
107559658152025736992729145688602,
3324154021716716493547315823808809,
106067493846954075776733869818571690,
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Example: $a_3 = 5$ because only one $\pi \in S_3$ is excluded.

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- The guessed operator has a right factor of order 8 and degree 11.
- We computed $a_{36}, a_{37}, a_{38}, a_{39}$ (!) and found that these terms were correctly predicted by the guessed recurrence.

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- It seems that this idea works at least on some examples
- Are there other features that can be used efficiently?