GERRYMANDERING

Manuel Kauers · Institute for Algebra · JKU

Joint work with Christoph Koutschan and George Spahn


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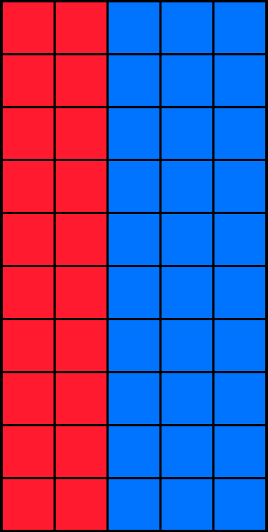
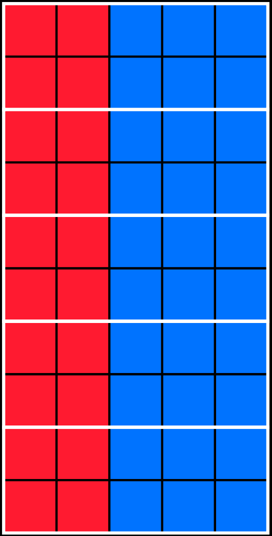
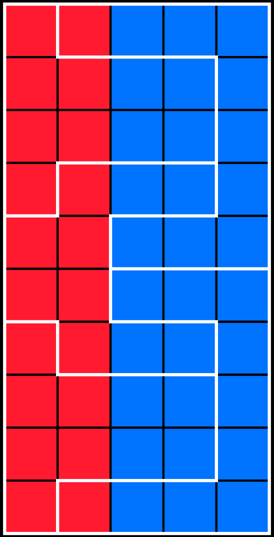
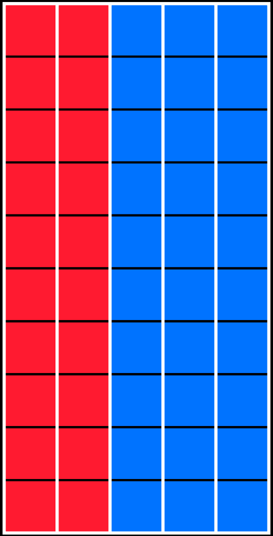
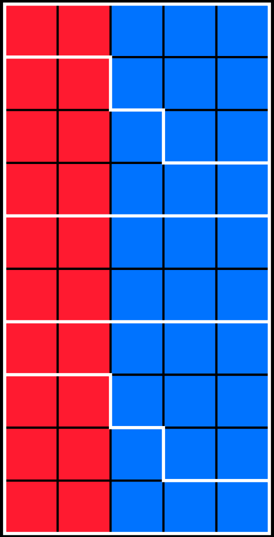
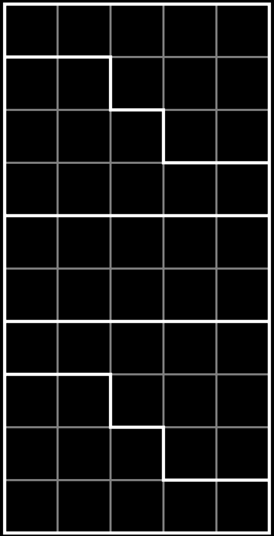


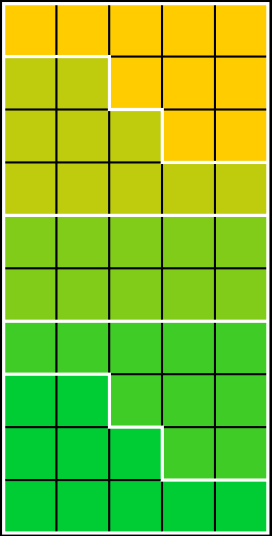


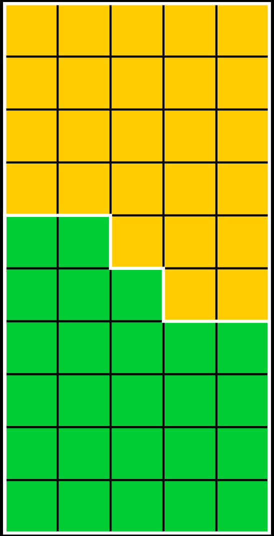


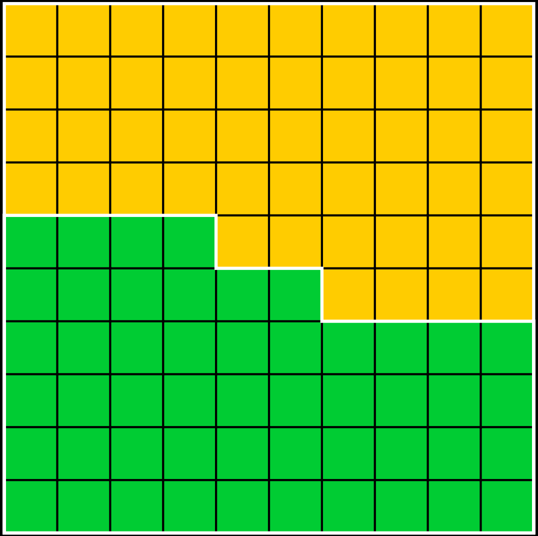












The OEIS is supported by the many generous donors to the OEIS Foundation.

0 1 3 6 2 7 13 20 12 21 11 22 10 23 OEIS THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A348456 Number of ways to dissect a 2*n X 2*n chessboard into two polyominoes each of area 2*n^2. 7

2, 70, 80518, 7157114189, 49852157614583644, 28289358593043414725944353, 1335056579423080371186456888543732162 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,1

COMMENTS See A348453 for much more information.

The board has 4*n^2 squares. The colors of the squares do not matter. The two parts are rook-connected polygons of area 2*n^2. They do not need to be the same polygon, only that they have the same area.

This is the "labeled" version of the problem. Symmetries of the square are not taken into account. Rotations and reflections count as different.

a(4) was found on May 04 2022 by George Spahn and Manuel Kauers using an 1838 X 1838 transfer matrix found by George Spahn (see the Zeilberger link). Manuel Kauers computed the [1,2] entry of the 9th power of that matrix. The desired number a(4) is half of the coefficient of z^32 in that entry. - Doron Zeilberger, May 04 2022

LINKS Table of n, a(n) for n=1..7.

Manuel Kauers, Christoph Koutschan, and George Spahn, A348456(4) = 7157114189, arXiv:2209.01787 [math.CO], 2022.

N. J. A. Sloane, The On-Line Encyclopedia of Integer Sequences: An illustrated guide with many unsolved problems, Guest Lecture given in Doron Zeilberger's Experimental Mathematics Math640 Class, Rutgers University, Spring Semester, Apr 28 2022: Slides; Slides (an alternative source).

Doron Zeilberger, Challenge to Manuel Kauers and his computer.

CROSSREFS A column of A348452 and A348453, and a diagonal of A348454 and A348455.

Sequence in context: A132566 A151686 A201555 * A293753 A164554 A141908

Adjacent sequences: A348453 A348454 A348455 * A348457 A348458 A348459

KEYWORD nonn,more

AUTHOR N. J. A. Sloane, Oct 27 2021

EXTENSIONS Added a(5)-a(7) (from the Kauers et al. reference), Joerg Arndt, Sep 07 2022

STATUS approved

Gerrymandering (2), cont.

T(k, d) = no. of ways to dissect a k X k square board into d rook-connected regions of size k^2 / d.

(A348452, A348456, A172477, A004003)

$k\backslash d$	1	2	3	4	5	6	7	8	9	$\dots$	
1	1										
2	1	2	0	1							
3	1	0	10^a	0	0	0	0	0	1		
4	1	70^b	0	117	0	0	0	36^c	0	$\dots$	$1_{@16}$
5	1	0	0	0	4096	0	0	0	0	$\dots$	$1_{@25}$
6	1	80518	264500	442791	0	451206	0	0	✓	$\dots$	$1_{@36}$
7	1	0	0	0	0	0	✓	0	0	$\dots$	$1_{@49}$
8	1	?	0	?	0	0	0	✓	0	$\dots$	$1_{@16}$

Most wanted: T(8,2) = no. of ways to cut chessboard into 2 rook-connected regions of area 32

Ignore colors of chessboard squares; rotations, reflections count as different; regions need not have same shape.

How large will T(8,2) be, roughly? How would you program it? How would you parallelize it?

Paul Zimmermann et al. in 2020 solved one of the RSA Challenge Problem,
It took them 2700 core years. How does T(8,2) compare?

Gerrymandering (2), cont. T(4,2) = 70:

Gerrymandering (2), cont. T(4,2) = 70:

70 WAYS TO DISSECT 4×4 BOARD

CODE BOUNDARY COUNT

2222 B=4 #=2
C=0 G=4
CORNERS GROUP ORDER
1

1133 B=6 #=4
C=2 G=2
2

1223 B=6 #=4
C=4 G=2
3

1313 B=10 #=4
C=6 G=2
4

1214 B=8 #=8
C=5 G=1
5

1322 B=7 #=8
C=4 G=1
6

1232 B=7 #=8
C=6 G=1
7

1114 B=7 #=8
C=3 G=1
8

1331 B=8 #=4
C=4 G=2
9

1124 B=6 #=4
C=3 G=2
10

1132 B=9 #=8
C=6 G=1
11

2132 B=8 #=4
C=6 G=2
12

1113 B=10 #=4
C=4 G=2
13

TOTAL = 70

Gerrymandering (2), cont. Tiling a Square with

Computational challenge for you (and your computer)
1 message

Doron Zeilberger <doronzeil@gmail.com> Wed, May 4, 2022 at 3:43 PM
To: Manuel Kauers <manuel@kauers.de>
Cc: George Spahn <gs828@math.rutgers.edu>, Neil Sloane <njasloane@gmail.com>

Dear Manuel,

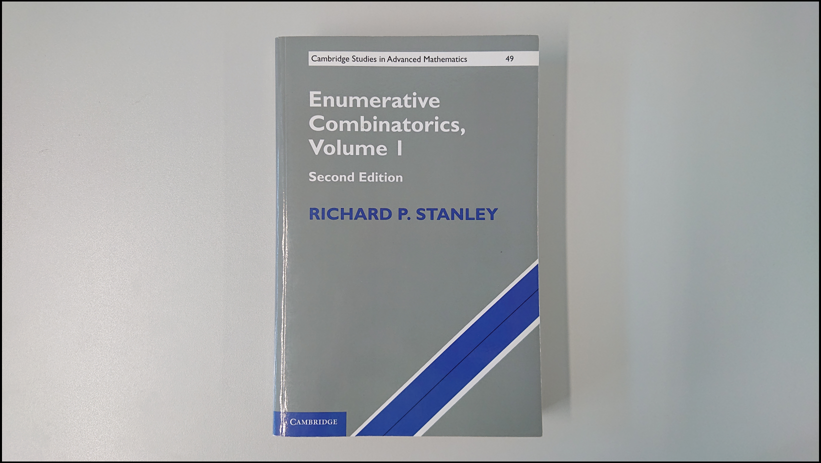
Hope you, Martina, and epsilon are doing well!

Can you (and your computers) meet the following challenge, in the secret url:https://sites.math.rutgers.edu/~zeilberg/EM22/C27.pdf

https://sites.math.rutgers.edu/~zeilberg/ChessChallenge.txt

If you do, I pledge to donate $100 to the OEIS in your honor.
Also, if you can do it systematically, this may lead to a joint paper with my student who can do other boards.

Best wishes,
Doron


Cambridge Studies in Advanced Mathematics 49
Enumerative Combinatorics, Volume 1
Second Edition
RICHARD P. STANLEY
CAMBRIDGE

4.7 The Transfer-Matrix Method

4.7.1 Basic Principles

The transfer-matrix method, like the Principle of Inclusion-Exclusion and the Möbius inversion formula, has simple theoretical underpinnings but a very wide range of applicability. The theoretical background can be divided into two parts—combinatorial and algebraic. First, we discuss the combinatorial part. A (finite) *directed graph* or *digraph* D is a triple (V,E,ϕ), where $V=\{v_1,\dots,v_p\}$ is a set of *vertices*, E is a finite set of (directed) *edges* or *arcs*, and ϕ is a map from E to $V\times V$. If $\phi(e)=(u,v)$, then e is called an edge *from u to v*, with *initial vertex* u and *final vertex* v. This is denoted $u=\operatorname{init} e$ and $v=\operatorname{fin} e$. If $u=v$, then e is called a *loop*. A *walk* Γ in D of *length* n from u to v is a sequence $e_1e_2\cdots e_n$ of n edges such that $\operatorname{init} e_1=u$, $\operatorname{fin} e_n=v$, and $\operatorname{fin} e_i=\operatorname{init} e_{i+1}$ for $1\le i<n$. If also $u=v$, then Γ is called a *closed walk based at u*. (Note that if Γ is a closed walk, then $e_ie_{i+1}\cdots e_ne_1\cdots e_{i-1}$ is in general a different closed walk. In some graph-theoretical contexts this distinction would not be made.)

Now let $w\colon E\to R$ be a *weight function* with values in some commutative ring R. (For our purposes here, we can take $R=\mathbb{C}$ or a polynomial ring over $\mathbb{C}$.) If $\Gamma=e_1e_2\cdots e_n$ is a walk, then the *weight* of Γ is defined by $w(\Gamma)=w(e_1)w(e_2)\cdots w(e_n)$. Let $i,j\in[p]$ and $n\in\mathbb{N}$. Since D is finite, we can define

$$A_{ij}(n)=\sum_{\Gamma} w(\Gamma),$$

where the sum is over all walks Γ in D of length n from v_i to v_j. In particular, $A_{ij}(0)=\delta_{ij}$. If all $w(e)=1$, then we are just counting the *number* of walks of length n from u to v. The fundamental problem treated by the transfer matrix method is the evaluation of $A_{ij}(n)$. The first step is to interpret $A_{ij}(n)$ as an entry in a certain matrix. Define a $p\times p$ matrix $A=(A_{ij})$ by

$$A_{ij}=\sum_{e} w(e),$$

where the sum ranges over all edges e satisfying $\operatorname{init} e=v_i$ and $\operatorname{fin} e=v_j$. In other words, $A_{ij}=A_{ij}(1)$. The matrix A is called the *adjacency matrix* of D, with respect to the weight function w. The eigenvalues of the adjacency matrix A play a key role in the enumeration of walks. These eigenvalues are also called the *eigenvalues of D* (as a weighted digraph).

4.7.1 Theorem. *Let $n\in\mathbb{N}$. Then the (i,j)-entry of A^n is equal to $A_{ij}(n)$. (Here we define $A^0=I$ even if A is not invertible.)*

Proof. The proof is immediate from the definition of matrix multiplication. Specifically, we have

$$(A^n)_{ij}=\sum A_{ii_1}A_{i_1i_2}\cdots A_{i_{n-1}j},$$

where the sum is over all sequences $(i_1,\dots,i_{n-1})\in[p]^{n-1}$. The summand is 0 unless there is a walk $e_1e_2\cdots e_n$ from v_i to v_j with $\operatorname{fin} e_k=v_{i_k}$ $(1\le k<n)$ and $\operatorname{init} e_k=v_{i_{k-1}}$ $(1<k\le n)$. If such a walk exists, then the summand is equal to the sum of the weights of all such walks, and the proof follows. □

The second step of the transfer-matrix method is the use of linear algebra to analyze the behavior of the function $A_{ij}(n)$. Define the generating function

$$F_{ij}(D,\lambda)=\sum_{n\ge 0} A_{ij}(n)\lambda^n.$$

4.7.2 Theorem. *The generating function $F_{ij}(D,\lambda)$ is given by*

$$F_{ij}(D,\lambda)=\frac{(-1)^{i+j}\det(I-\lambda A:j,i)}{\det(I-\lambda A)}, \tag{4.34}$$

where $(B:j,i)$ denotes the matrix obtained by removing the j-th row and i-th column of B. Thus in particular $F_{ij}(D,\lambda)$ is a rational function of λ whose degree is strictly less than the multiplicity n_0 of 0 as an eigenvalue of A.

Proof. $F_{ij}(D,\lambda)$ is the (i,j)-entry of the matrix $\sum_{n\ge0}\lambda^nA^n=(I-\lambda A)^{-1}$. If B is any invertible matrix, then it is well known from linear algebra that $(B^{-1})_{ij}=(-1)^{i+j}\det(B:j,i)/\det(B)$, so equation (4.34) follows.

Suppose now that A is a $p\times p$ matrix. Then

$$\det(I-\lambda A)=1+\alpha_1\lambda+\cdots+\alpha_{p-n_0}\lambda^{p-n_0},$$

where

$$(-1)^p\left(\alpha_{p-n_0}\lambda^{n_0}+\cdots+\alpha_1\lambda^{p-1}+\lambda^p\right)$$

is the characteristic polynomial $\det(A-\lambda I)$ of A. Thus as polynomials in λ, we have $\deg\det(I-\lambda A)=p-n_0$ and $\deg\det(I-\lambda A:j,i)\le p-1$. Hence,

$$\deg F_{ij}\le p-1-(p-n_0)<n_0. \qquad \square$$

One special case of Theorem 4.7.2 is particularly elegant. Let

$$C_D(n)=\sum_{\Gamma} w(\Gamma),$$

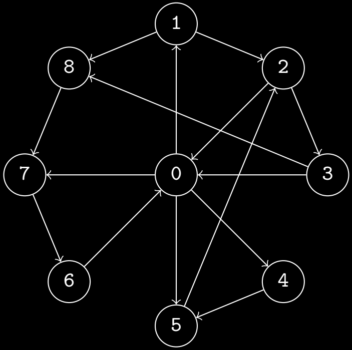
where the sum is over all closed walks Γ in D of length n. For instance, $C_D(1)=\operatorname{tr} A$, where tr denotes trace.

4.7.3 Corollary. *Let $Q(\lambda)=\det(I-\lambda A)$. Then*

$$\sum_{n\ge1} C_D(n)\lambda^n=-\frac{\lambda Q'(\lambda)}{Q(\lambda)}.$$

Proof. By Theorem 4.7.1, we have

$$C_D(n)=\sum_{i=1}^{p} A_{ii}(n)=\operatorname{tr} A^n.$$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$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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3 & 4 & 0 & 1 & 4 & 4 & 1 & 4 & 2 \\
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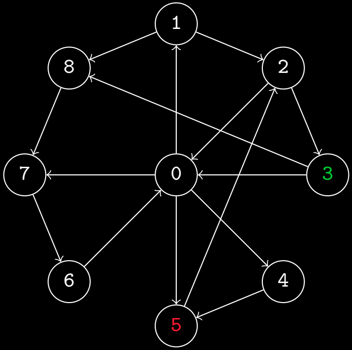
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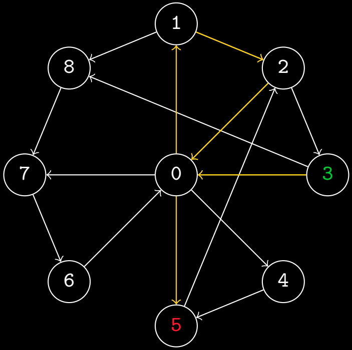
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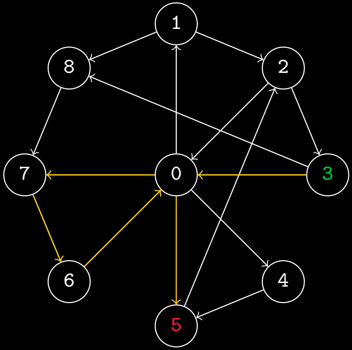
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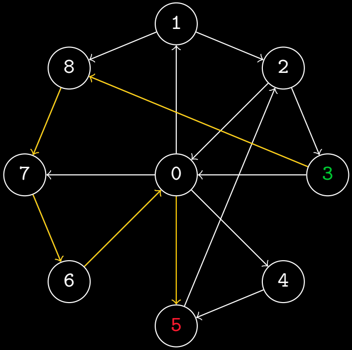
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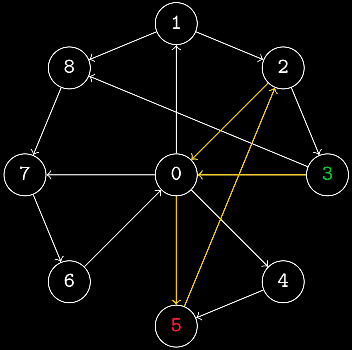
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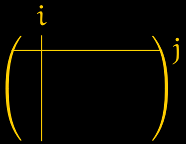
$$\sum_{n=0}^{\infty} A_{i,j}(n)x^n = (-1)^{i+j}\frac{\det(I_k - x\,A)^{[j,i]}}{\det(I_k - x\,A)}$$

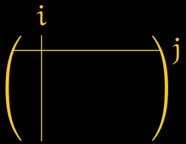
$$\sum_{n=0}^{\infty} \underbrace{A_{i,j}(n)}_{\substack{\#\text{ paths with}\\ n\text{ steps from}\\ \text{vertex } i\text{ to}\\ \text{vertex } j}} x^n = (-1)^{i+j} \frac{\det(I_k - x\,A)^{[j,i]}}{\det(I_k - x\,A)}$$

$$\sum_{n=0}^{\infty} \underbrace{A_{i,j}(n)}_{\substack{\#\text{ paths with}\\ n\text{ steps from}\\ \text{vertex } i \text{ to}\\ \text{vertex } j}} x^n = (-1)^{i+j} \frac{\det(I_k - x\,A)^{[j,i]}}{\det(I_k - x\,\underset{\substack{\uparrow\\ \text{adjacency}\\ \text{matrix}}}{A})}$$



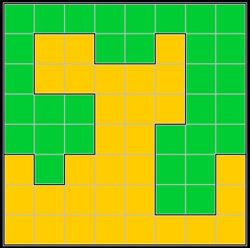
$$\sum_{n=0}^{\infty} \underbrace{A_{i,j}(n)}_{\substack{\#\text{ paths with}\\ n\text{ steps from}\\ \text{vertex } i \text{ to}\\ \text{vertex } j}} x^n = (-1)^{i+j} \frac{\det(I_k - x\,A)^{[j,i]}}{\det(I_k - x\,A)}$$

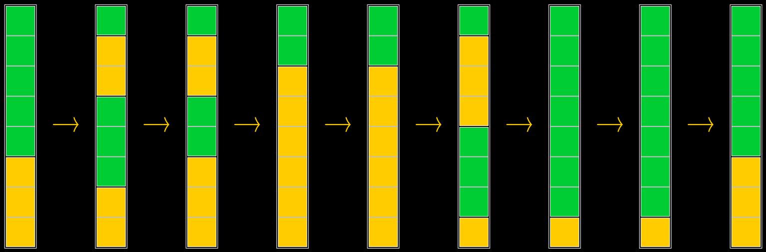
adjacency
matrix

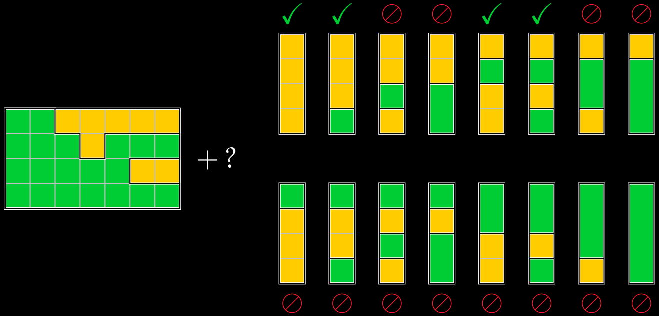


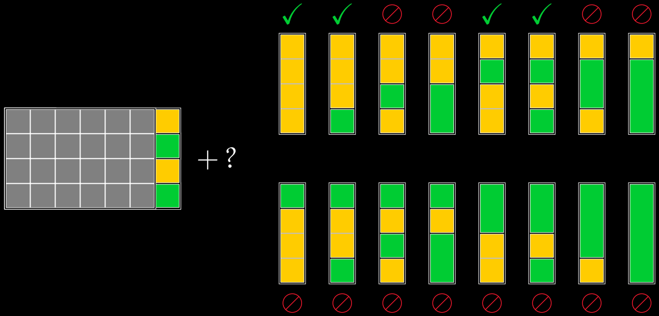
$$\sum_{n=0}^{\infty} \underbrace{A_{i,j}(n)}_{\substack{\#\ \text{paths with} \\ n\ \text{steps from} \\ \text{vertex } i \text{ to} \\ \text{vertex } j}} x^n = (-1)^{i+j}\frac{\det(I_k - x\,A)^{[j,i]}}{\det(I_k - x\,A)} \in \mathbb{Q}(x)$$

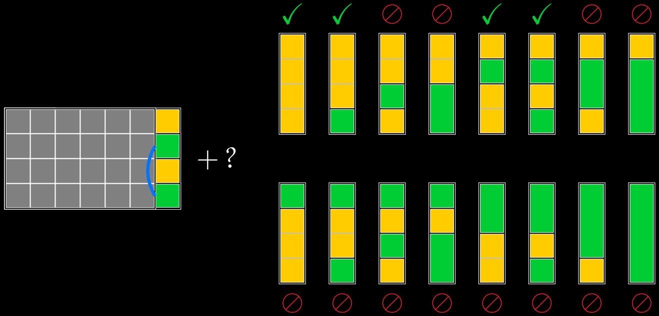
adjacency
matrix

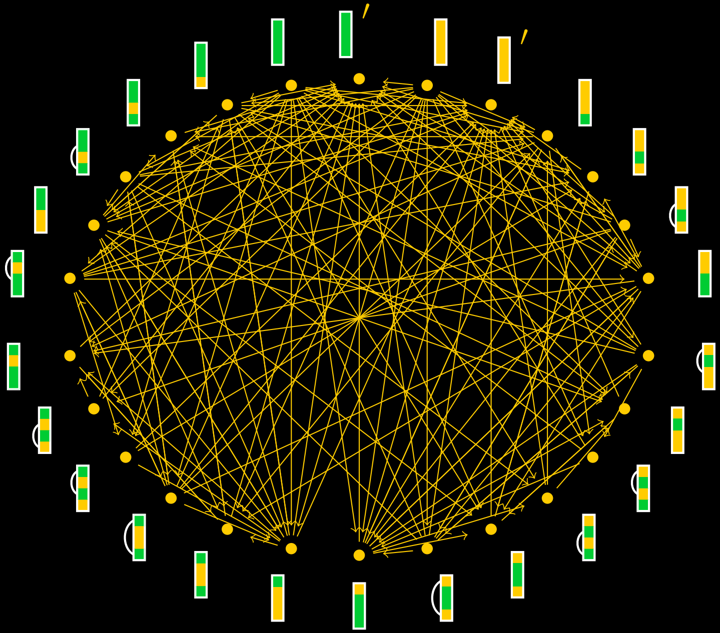


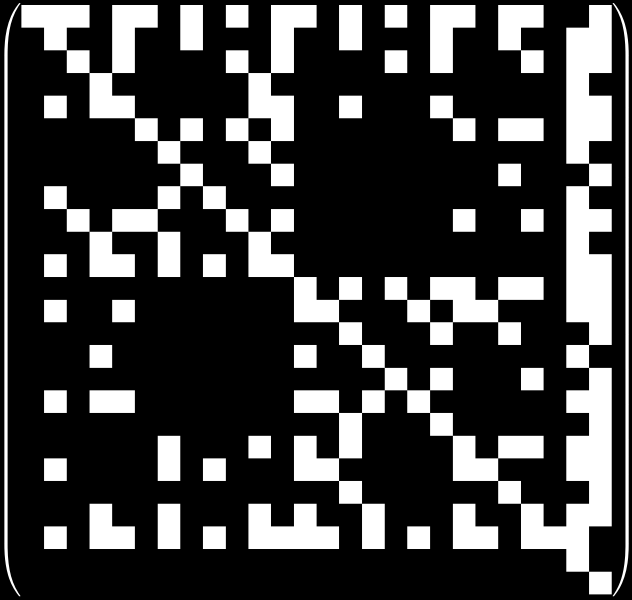


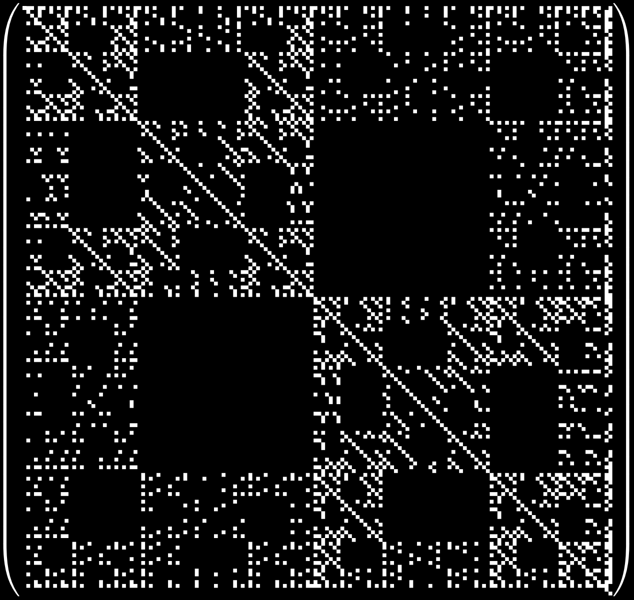








$A =$ 

$A =$ 

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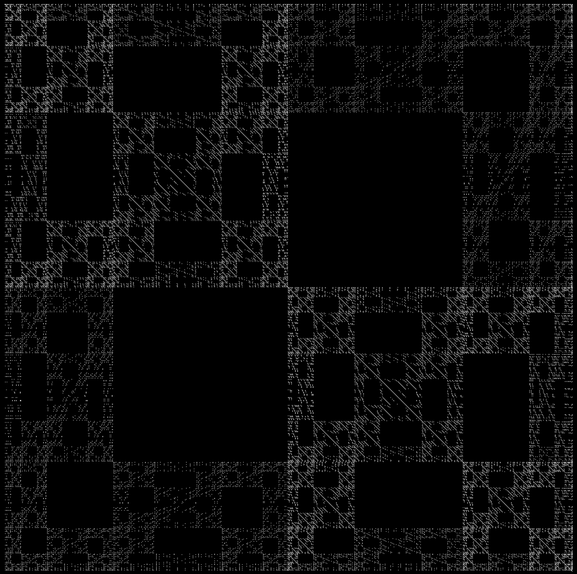

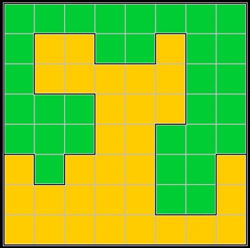

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0 & 0 & 0 & z^3 & 0 & 0 & 0 & 0 & 0 & 0 & z^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z^4 & 0 \\
0 & z^3 & 0 & z^3 & z^2 & 0 & 0 & 0 & 0 & 0 & z^2 & z & 0 & 0 & z^2 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & 0 & z^4 & z^0 \\
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0 & z^3 & 0 & z^3 & z^2 & 0 & z^3 & 0 & z^2 & 0 & z^2 & z & z^3 & z^2 & 0 & z^2 & 0 & z & 0 & z^2 & z & 0 & z & z^0 & z^4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z^4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z^0
\end{pmatrix}
$$

$$\text{gerrymander}(2n \times 2n) = \frac{1}{2}[z^{2n^2}]\left(v_{\text{init}}\, A^{2n-1}\, v_{\text{final}}\right)$$

$$\mathsf{gerrymander}(2n \times 2n) = \frac{1}{2}[z^{2n^2}]\big(v_{\mathsf{init}}\, A^{2n-1}\, v_{\mathsf{final}}\big)$$

$$\mathsf{gerrymander}(8 \times 8) = 7157114189$$

$$\text{gerrymander}(2n \times 2n) = \frac{1}{2}[z^{2n^2}]\left(v_{\text{init}}\, A^{2n-1}\, v_{\text{final}}\right)$$

$$\begin{aligned}
\text{gerrymander}(8 \times 8) &= 7157114189 \\
\text{gerrymander}(10 \times 10) &= 49852157614583644 \\
\text{gerrymander}(12 \times 12) &= 28289358593043414725944353 \\
\text{gerrymander}(14 \times 14) &= 13350565794230803711864568 \\
& 88543732162
\end{aligned}$$

Gerrymander sequence or A348456 - mkauers@gmail.com - Gmail — Mozilla Firefox

https://mail.google.com/mail/u/0/popout?ver=1imoqlq44gn1z&search=inbox&type=WhctKKXpMTsv(133%

Gerrymander sequence or A348456

Tony Guttmann tony.guttmann@gmail... 1:35 AM (9 hours ago)
to Doron, geosp98, Neil, manuel, christoph.koutschan, IWAN

Gentlemen,

We were fascinated to read the recent arXiv posting by Manuel, Christoph and George, and then watch the seminar by Neil, with Doron's comments.

We thought we had the technology to improve on your enumerations, and indeed this proved to be so. We have 4 further terms.

Perhaps more significantly we've proved the connection to the well-known problem of self-avoiding walks crossing a square (either from corner-to-corner or
transversely). Thus we can give the (conjectured) asymptotics of gerrymanders to quite high precision, and can provably give the general form.

The draft attached is going onto the arXiv today. We welcome any comments or corrections, praise or abuse.

$$
\begin{aligned}
\text{gerrymander}(16 \times 16) &= 5288157175943649955880910\\
&\ 966508435029578848399795\\
\text{gerrymander}(18 \times 18) &= 1768514227824943648668138\\
&\ 1532269984302096268367750\\
&\ 21539911012000\\
\text{gerrymander}(20 \times 20) &= 5012626198719413833309526\\
&\ 6040242179892262270498222\\
&\ 2422277677102771194891941\\
&\ 26252\\
\text{gerrymander}(22 \times 22) &= 1207270800266539956834051\\
&\ 0850610912278859297261103\\
&\ 5310673809853406496349171\\
&\ 003311517916839962975062
\end{aligned}
$$

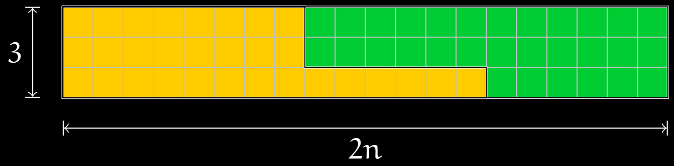
$$\text{gerrymander}(2n \times 2n) \sim c\,\lambda^{n^2} \quad (n \to \infty)$$

$$\text{gerrymander}(2n \times 2n) \sim c\,\lambda^{n^2} \quad (n \to \infty)$$

not D-finite.

11929 [2016, 831]. *Proposed by Donald Knuth, Stanford University, Stanford, CA.* Let a_n be the number of ways in which a rectangular box that contains $6n$ square tiles in three rows of length $2n$ can be split into two connected pieces of size $3n$ without cutting any tiles. Thus $a_1 = 3$, $a_2 = 19$, and one of the 85 ways for $n = 3$ is shown.

Taking $a_0 = 1$, find a closed form for the generating function $A(z) = \sum_{n=0}^{\infty} a_n z^n$. What is the asymptotic nature of a_n as $n \to \infty$?


3
2n

$$\frac{1}{2}[z^0]\sum_{i,j}(-1)^{i+j}\frac{\det(I_k - xz^{-3}A^{[j,i]})}{\det(I_k - xz^{-3}A)}$$

$$\frac{1}{2}[z^0]\sum_{i,j}(-1)^{i+j}\frac{\det(I_k - xz^{-3}A^{[j,i]})}{\det(I_k - xz^{-3}A)}$$

algebraic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OFFSET	0,4
LINKS	Alois P. Heinz, Table of n, a(n) for n = 0..40
CROSSREFS	Main diagonal of A323846. Column d=2 of A323848. Sequence in context: A228216 A261485 A125482 * A344733 A028130 A286719 Adjacent sequences: A306319 A306320 A306321 * A306323 A306324 A306325
KEYWORD	nonn
AUTHOR	Alois P. Heinz, Feb 07 2019
STATUS	approved

0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	0	1	1	1	1
0	0	0	1	1	1	1
0	0	0	1	2	2	2
0	0	1	1	2	2	2
0	1	1	2	2	2	2

$$
\begin{array}{|c|}\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline\end{array}
\longrightarrow
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\longrightarrow
\begin{array}{|c|}\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline 0\\\hline 1\\\hline 1\\\hline\end{array}
\longrightarrow
\begin{array}{|c|}\hline 0\\\hline 0\\\hline 1\\\hline 1\\\hline 1\\\hline 1\\\hline 2\\\hline\end{array}
\longrightarrow
\begin{array}{|c|}\hline 0\\\hline 0\\\hline 1\\\hline 1\\\hline 2\\\hline 2\\\hline 2\\\hline\end{array}
\longrightarrow
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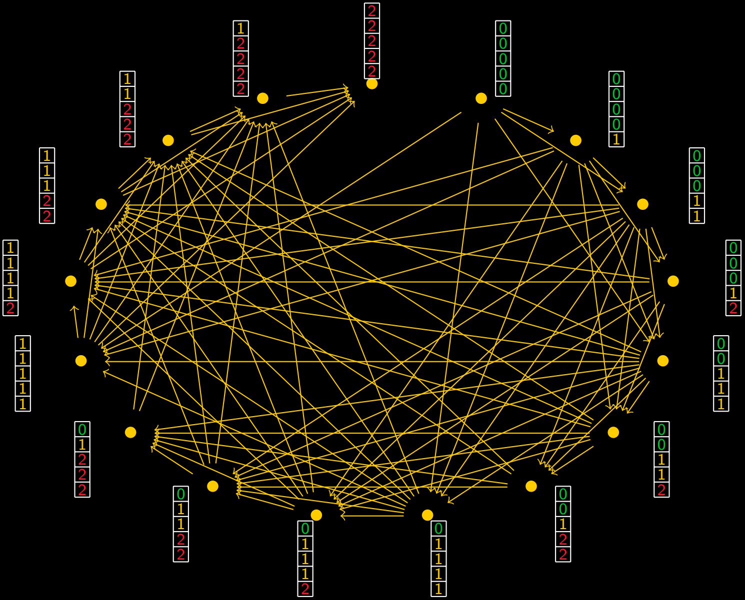
\begin{array}{|c|}\hline 0\\\hline 1\\\hline 1\\\hline 1\\\hline 2\\\hline 2\\\hline 2\\\hline\end{array}
$$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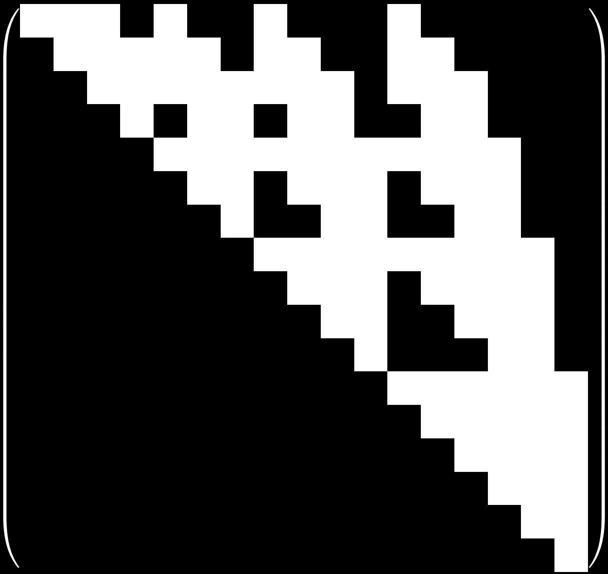
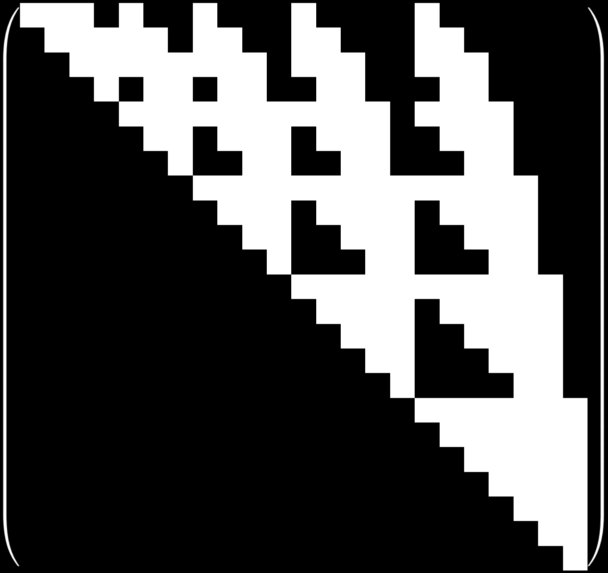
$A =$ 

$A =$ 

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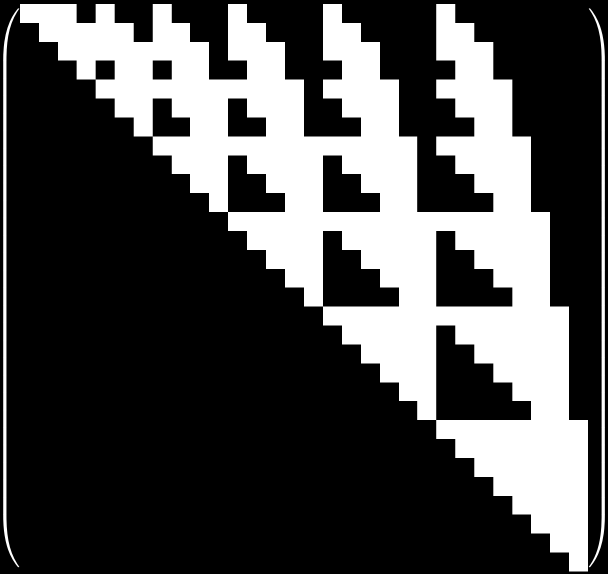

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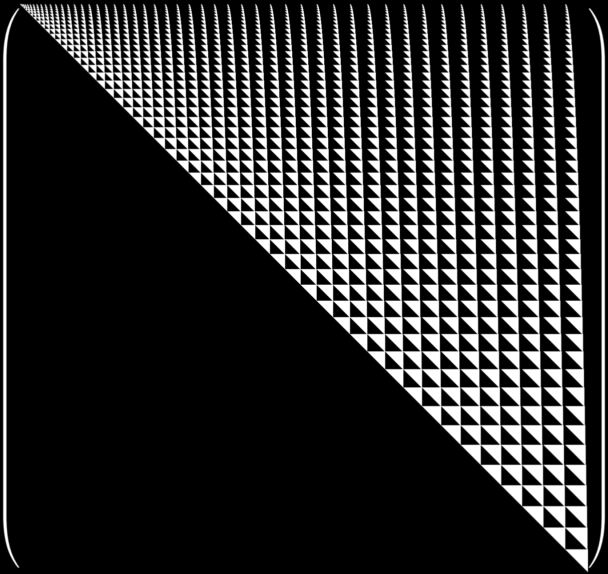


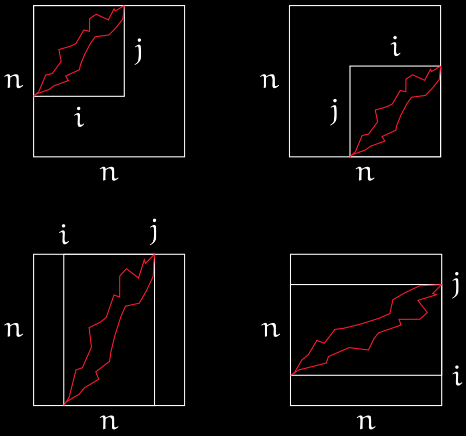
	1	2	3	4	5	6	7
1	0	0	1	3	6	10	15
2	0	0	4	16	41	85	155
3	1	4	25	94	266	632	1332
4	3	16	94	386	1247	3423	8342
5	6	41	266	1247	4657	14795	41586
6	10	85	632	3423	14795	54219	174844
7	15	155	1332	8342	41586	174844	642815
8	21	259	2570	18546	106067	508484	2117690
9	28	406	4631	38304	249814	1357051	6362806
10	36	606	7900	74451	550334	3367166	17671203

	1	2	3	4	5	6	7
1	0	0	1	3	6	10	15
2	0	0	4	16	41	85	155
3	1	4	25	94	266	632	1332
4	3	16	94	386	1247	3423	8342
5	6	41	266	1247	4657	14795	41586
6	10	85	632	3423	14795	54219	174844
7	15	155	1332	8342	41586	174844	642815
8	21	259	2570	18546	106067	508484	2117690
9	28	406	4631	38304	249814	1357051	6362806
10	36	606	7900	74451	550334	3367166	17671203

0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	0	1	1	1	1
0	0	0	1	1	1	1
0	0	0	1	2	2	2
0	0	1	1	2	2	2
0	1	1	2	2	2	2

0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	0	1	1	1	1
0	0	0	1	1	1	1
0	0	0	1	2	2	2
0	0	1	1	2	2	2
0	1	1	2	2	2	2

$$N_{i,j} = \frac{1}{i+j-1}\binom{i+j-1}{i}\binom{i+j-1}{i-1}$$



n
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$$2\sum_{j=1}^{n}\left(\sum_{i=1}^{n} N_{i,j} + (n-j-1)N_{j,n}\right) - 2\binom{2n}{n} + N_{n,n} + 3$$

$$2\sum_{j=1}^{n}\left(\sum_{i=1}^{n} N_{i,j} + (n-j-1)N_{j,n}\right) - 2\binom{2n}{n} + N_{n,n} + 3$$

D-finite.

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0 1 3 6 2 7 13 20 12 21 11 22 10 23 OEIS

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KEYWORD	nonn
AUTHOR	Alois P. Heinz, Feb 07 2019
STATUS	approved

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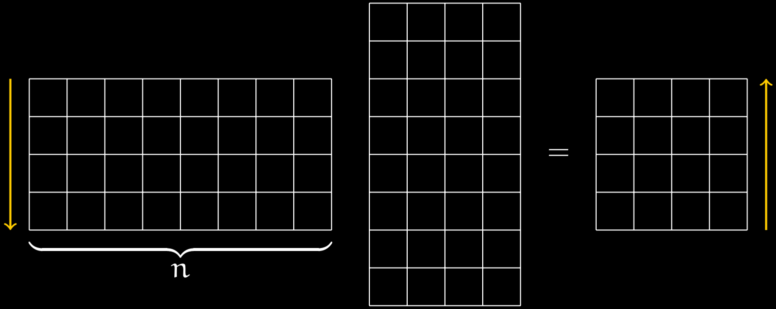
A181280 Number of 4Xn binary matrices M with rows in strictly increasing order and rows of M*Mtranspose (mod 2) in strictly decreasing order 1

0, 0, 0, 58, 1629, 28924, 507052, 8211776, 133693904, 2140571200, 34361115072, 549587348992, 8798356254976, 140744002571264, 2252082614856704, 36030315649662976, 576487656686899200, 9223539637335310336 (list; graph; refs; listen; history; text; internal format)

OFFSET	1,4
COMMENTS	Row 4 of A181274
LINKS	R. H. Hardin, Table of n, a(n) for n=1..27
EXAMPLE	M and M*Mtranspose (mod 2) for 4X5 ..0..1..0..1..1......1..0..0..0 ..1..0..0..0..0......0..1..1..1 ..1..1..0..0..1......0..1..1..0 ..1..1..1..1..0......0..1..0..0
CROSSREFS	Sequence in context: A093258 A184599 A160347 * A017774 A035724 A017721 Adjacent sequences: A181277 A181278 A181279 * A181281 A181282 A181283
KEYWORD	nonn
AUTHOR	R. H. Hardin Oct 10 2010
STATUS	approved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$$\begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \cdot & \bullet & \cdot & \cdot \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \cdot & \bullet & \cdot & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \bullet & \cdot & \cdot & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \cdot & \bullet & \cdot \\ \bullet & \cdot & \bullet & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix},$$

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$$\begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \cdot \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \cdot \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \cdot & \bullet & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \cdot \\ \bullet & \cdot & \cdot & \cdot \\ \bullet & \cdot & \bullet & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \bullet \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \bullet \\ \cdot & \bullet & \bullet & \cdot \\ \bullet & \cdot & \bullet & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \cdot & \bullet \\ \bullet & \cdot & \cdot & \cdot \\ \bullet & \cdot & \bullet & \cdot \end{pmatrix},$$

$$\begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \cdot & \cdot & \bullet \\ \bullet & \cdot & \bullet & \bullet \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \cdot & \cdot & \bullet \\ \bullet & \bullet & \cdot & \bullet \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \cdot & \bullet & \bullet \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \cdot & \bullet & \bullet \\ \bullet & \bullet & \cdot & \bullet \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \cdot & \bullet & \bullet \\ \bullet & \bullet & \bullet & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \cdot & \bullet & \bullet & \bullet \\ \bullet & \bullet & \cdot & \bullet \\ \bullet & \bullet & \bullet & \cdot \end{pmatrix},$$

$$\begin{pmatrix} \cdot & \cdot & \cdot & \bullet \\ \bullet & \cdot & \bullet & \bullet \\ \bullet & \bullet & \cdot & \bullet \\ \bullet & \bullet & \bullet & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet & \bullet \\ \cdot & \bullet & \cdot & \cdot \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet & \bullet \\ \cdot & \bullet & \cdot & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet & \bullet \\ \cdot & \bullet & \cdot & \bullet \\ \bullet & \cdot & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet & \bullet \\ \cdot & \bullet & \cdot & \bullet \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet & \bullet \\ \bullet & \cdot & \cdot & \cdot \\ \bullet & \bullet & \cdot & \cdot \end{pmatrix},$$

$$\begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\cdot&\bullet&\bullet\\\bullet&\cdot&\cdot&\bullet\\\bullet&\bullet&\cdot&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\cdot&\bullet\\\bullet&\cdot&\cdot&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\cdot&\bullet\\\bullet&\cdot&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\cdot&\cdot\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\cdot\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\cdot\\\bullet&\cdot&\bullet&\bullet\end{pmatrix},$$

$$\begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\cdot\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\cdot\\\bullet&\bullet&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\cdot&\bullet&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\cdot&\bullet\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\cdot&\bullet\\\bullet&\cdot&\cdot&\bullet\\\bullet&\cdot&\bullet&\cdot\end{pmatrix},$$

$$\begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\bullet&\cdot\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\cdot&\cdot\\\cdot&\bullet&\bullet&\bullet\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix},$$

$$\begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\cdot&\cdot\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\cdot&\bullet\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\cdot\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\cdot&\bullet&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix},$$

$$\begin{pmatrix}\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\cdot&\bullet\end{pmatrix}, \begin{pmatrix}\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\bullet&\cdot&\cdot&\cdot\\\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}, \begin{pmatrix}\bullet&\cdot&\bullet&\bullet\\\bullet&\bullet&\cdot&\cdot\\\bullet&\bullet&\cdot&\bullet\\\bullet&\bullet&\bullet&\cdot\end{pmatrix}.$$

$0, 0, 0, 58, 1629, 28924, 507052, 8211776, 133693904, \ldots$

$$-22912660668416(n-2)a(n)+4194304(4419089n-5784790)a(n+1)+458752(9499785n-97594504)a(n+2)-8192(890967049n-4679365255)a(n+3)+1024(1488027923n-5601351692)a(n+4)+6528(44221759n-387235809)a(n+5)-32(3992176883n-26858644798)a(n+6)+8(1107194741n-6399743425)a(n+7)+2(610453317n-4593544888)a(n+8)-(182139823n-1273140745)a(n+9)+(6061186n-41678719)a(n+10)\stackrel{?}{=}0$$

?

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(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A269021 Number of permutations of [2n] containing an increasing subsequence of length n. 3

1, 2, 23, 588, 24553, 1438112, 108469917, 9996042284, 1086997811325, 136102249609224, 19269396089593156, 3042212958893941456, 529708789768374664407, 100813134967124531098768, 20816198414187782633783462, 4634136282168760818748363080 (list; graph; refs; listen; history; text; internal format)

OFFSET	0,2
LINKS	Alois P. Heinz and Vaclav Kotesovec, Table of n, a(n) for n = 0..41 (terms 0..30 from Alois P. Heinz)
FORMULA	a(n) = A214152(2n,n). a(n) = (2n)! - A269042(n). a(n) ~ 16^n * (n-1)! / (Pi * exp(2)). - Vaclav Kotesovec, Mar 27 2016
EXAMPLE	a(1) = 2: 12, 21. a(2) = 23: all 4! permutations of {1,2,3,4} with the exception of 4321.

MAPLE
```
h:= proc(l) (n-> add(i, i=l)!/mul(mul(1+l[i]-j+add(`if`(
      l[k]>=j, 1, 0), k=i+1..n), j=1..l[i]), i=1..n))(nops(l))
    end:
g:= (n, i, l)-> `if`(n=0 or i=1, h([l[], 1$n])^2, `if`(i<1, 0,
                add(g(n-i*j, i-1, [l[], i$j]), j=0..n/i))):
a:= n-> `if`(n=0, 1, (2*n)!-g(2*n, n-1, [])):
seq(a(n), n=0..16);
```

MATHEMATICA
```
h[l_] := Function[n, Total[l]!/Product[Product[1+l[[i]]-j+Sum[If[l[[k]] >= j, 1,
   0], {k, i+1, n}], {j, 1, l[[i]]}], {i, 1, n}]][Length[l]];
g[n_, i_, l_] := If[n == 0 || i == 1, h[Join[l, Table[1, {n}]]]^2, If[i < 1, 0,
   Sum[g[n - i*j, i - 1, Join[l, Table[i, {j}]]], {j, 0, n/i}]]];
a[n_] := If[n == 0, 1, (2n)! - g[2n, n-1, {}]];
Table[a[n], {n, 0, 16}] (* Jean-François Alcover, Apr 01 2017, translated from
   Maple *)
```

CROSSREFS	Cf. A010050, A214152, A269042. Sequence in context: A084322 A073062 A015098 * A136039 A237580 A273976 Adjacent sequences: A269018 A269019 A269020 * A269022 A269023 A269024
KEYWORD	nonn
AUTHOR	Alois P. Heinz, Feb 17 2016
STATUS	approved

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 2 & 10 & 4 & 6 & 1 & 8 & 3 & 7 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 2 & 10 & 4 & 6 & 1 & 8 & 3 & 7 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 2 & 10 & 4 & 6 & 1 & 8 & 3 & 7 & 9 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 2 & 10 & 4 & 6 & 1 & 8 & 3 & 7 & 9 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 9 & 4 & 7 & 6 & 3 & 8 & 5 & 2 & 1 \end{pmatrix} \times$$

123456, 123465, 123546, 123564, 123645, 123654, 124356, 124365, 124536, 124563, 124635, 124653, 125346,
125364, 125436, 125463, 125634, 125643, 126345, 126354, 126435, 126453, 126534, 126543, 132456, 132465,
132546, 132564, 132645, 132654, 134256, 134265, 134526, 134562, 134625, 134652, 135246, 135264, 135426,
135462, 135624, 135642, 136245, 136254, 136425, 136452, 136524, 136542, 142356, 142365, 142536, 142563,
142635, 142653, 143256, 143265, 143526, 143562, 143625, 143652, 145236, 145263, 145326, 145362, 145623,
145632, 146235, 146253, 146325, 146352, 146523, 146532, 152346, 152364, 152436, 152463, 152634, 152643,
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165342, 165423, 165432, 213456, 213465, 213546, 213564, 213645, 213654, 214356, 214365, 214536, 214563,
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324561, 324615, 324651, 325146, 325164, 325416, 325461, 325614, 325641, 326145, 326154, 326415, 326451,
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354162, 354216, 354261, 354612, 354621, 356124, 356142, 356214, 356241, 356412, 356421, 361245, 361254,
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364251, 364512, 364521, 365124, 365142, 365214, 365241, 365412, 365421,

412356, 412365, 412536, 412563, 412635, 412653, 413256, 413265, 413526, 413562, 413625, 413652, 415236, 415263, 415326, 415362, 415623, 415632, 416235, 416253, 416325, 416352, 416523, 416532, 421356, 421365, 421536, 421563, 421635, 421653, 423156, 423165, 423516, 423561, 423615, 423651, 425136, 425163, 425316, 425361, 425613, 425631, 426135, 426153, 426315, 426351, 426513, 426531, 431256, 431265, 431526, 431562, 431625, 431652, 432156, 432165, 432516, 432561, 432615, 432651, 435126, 435162, 435216, 435261, 435612, 435621, 436125, 436152, 436215, 436251, 436512, 436521, 451236, 451263, 451326, 451362, 451623, 451632, 452136, 452163, 452316, 452361, 452613, 452631, 453126, 453162, 453216, 453261, 453612, 453621, 456123, 456132, 456213, 456231, 456312, 456321, 461235, 461253, 461325, 461352, 461523, 461532, 462135, 462153, 462315, 462351, 462513, 462531, 463125, 463152, 463215, 463251, 463512, 463521, 465123, 465132, 465213, 465231, 465312, 465321, 512346, 512364, 512436, 512463, 512634, 512643, 513246, 513264, 513426, 513462, 513624, 513642, 514236, 514263, 514326, 514362, 514623, 514632, 516234, 516243, 516324, 516342, 516423, 516432, 521346, 521364, 521436, 521463, 521634, 521643, 523146, 523164, 523416, 523461, 523614, 523641, 524136, 524163, 524316, 524361, 524613, 524631, 526134, 526143, 526314, 526341, 526413, 526431, 531246, 531264, 531426, 531462, 531624, 531642, 532146, 532164, 532416, 532461, 532614, 532641, 534126, 534162, 534216, 534261, 534612, 534621, 536124, 536142, 536214, 536241, 536412, 536421, 541236, 541263, 541326, 541362, 541623, 541632, 542136, 542163, 542316, 542361, 542613, 542631, 543126, 543162, 543216, 543261, 543612, 543621, 546123, 546132, 546213, 546231, 546312, 546321, 561234, 561243, 561324, 561342, 561423, 561432, 562134, 562143, 562314, 562341, 562413, 562431, 563124, 563142, 563214, 563241, 563412, 563421, 564123, 564132, 564213, 564231, 564312, 564321, 612345, 612354, 612435, 612453, 612534, 612543, 613245, 613254, 613425, 613452, 613524, 613542, 614235, 614253, 614325, 614352, 614523, 614532, 615234, 615243, 615324, 615342, 615423, 615432, 621345, 621354, 621435, 621453, 621534, 621543, 623145, 623154, 623415, 623451, 623514, 623541, 624135, 624153, 624315, 624351, 624513, 624531, 625134, 625143, 625314, 625341, 625413, 625431, 631245, 631254, 631425, 631452, 631524, 631542, 632145, 632154, 632415, 632451, 632514, 632541, 634125, 634152, 634215, 634251, 634512, 634521, 635124, 635142, 635214, 635241, 635412, 635421, 641235, 641253, 641325, 641352, 641523, 641532, 642135, 642153, 642315, 642351, 642513, 642531, 643125, 643152, 643215, 643251, 643512, 643521, 645123, 645132, 645213, 645231, 645312, 645321, 651234, 651243, 651324, 651342, 651423, 651432, 652134, 652143, 652314, 652341, 652413, 652431, 653124, 653142, 653214, 653241, 653412, 653421, 654123, 654132, 654213, 654231, 654312, 654321.

$1, 2, 23, 588, 24553, 1438112, 108469917, 9996042284, \ldots$

$$a(n) = (2n)! - \mathsf{Av}(123\ldots n, 2n)$$

$$
\begin{gathered}
(-1728n^{13} - 53136n^{12} - 721140n^{11} - 5712687n^{10} - 29351094n^{9} - 102460062n^{8} - \\
246409440n^{7} - 402006033n^{6} - 417533082n^{5} - 219507006n^{4} + 31431384n^{3} + 124878744n^{2} + \\
72946080n + 15264000)a(n+4) + (1024n^{16} + 53504n^{15} + 1249600n^{14} + 17355888n^{13} + \\
160521192n^{12} + 1047437752n^{11} + 4977166762n^{10} + 17495132870n^{9} + 45641586148n^{8} + \\
87532009076n^{7} + 120008214078n^{6} + 110095871882n^{5} + 55345804604n^{4} - 1557439740n^{3} - \\
21624996480n^{2} - 12720199680n - 2600484000)a(n+3) + (-8192n^{18} - 410624n^{17} - \\
9642752n^{16} - 140657088n^{15} - 1423927920n^{14} - 10587567456n^{13} - 59702792428n^{12} - \\
260014605984n^{11} - 882548274548n^{10} - 2338765850568n^{9} - 4813195967836n^{8} - \\
7586276752416n^{7} - 8912326817932n^{6} - 7398938790792n^{5} - 3819315829640n^{4} - \\
665124205264n^{3} + 522524456960n^{2} + 369006456000n + 75140208000)a(n+2) + (16384n^{20} + \\
929792n^{19} + 24570880n^{18} + 401745152n^{17} + 4556681984n^{16} + 38100385152n^{15} + \\
243590345536n^{14} + 1218638399056n^{13} + 4840529161808n^{12} + 15394386577552n^{11} + \\
39333747617456n^{10} + 80646042536528n^{9} + 131878942584240n^{8} + 169944390391856n^{7} + \\
169071055038224n^{6} + 125450305626848n^{5} + 65194277144320n^{4} + 20586079186752n^{3} + \\
2099910113280n^{2} - 851205715200n - 239459328000)a(n+1) + (-262144n^{21} - \\
12779520n^{20} - 289914880n^{19} - 4076081152n^{18} - 39877844992n^{17} - 288808008704n^{16} - \\
1607526882304n^{15} - 7044835693312n^{14} - 24696634639360n^{13} - 69962924768064n^{12} - \\
161099262705152n^{11} - 302196850871360n^{10} - 461330891824128n^{9} - 570677114273600n^{8} - \\
567515326534144n^{7} - 448149882466496n^{6} - 275971184318976n^{5} - 129065105827712n^{4} - \\
44053232273920n^{3} - 10293302100480n^{2} - 1463684198400n - 94998528000)a(n) \stackrel{?}{=} 0
\end{gathered}
$$

?

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Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A164735 Number of n-digit cycles of length 3 under the Kaprekar map A151949 6

0, 0, 0, 0, 0, 0, 0, 1, 0, 4, 0, 10, 0, 20, 0, 36, 0, 60, 1, 94, 4, 141, 10, 204, 21, 286, 39, 392, 66, 527, 105, 696, 159, 906, 231, 1164, 326, 1477, 449, 1854, 605, 2304, 801, 2836, 1044, 3462, 1341, 4194, 1701, 5044, 2133, 6027, 2646, 7158, 3252, 8452, 3963 (list; graph; refs; listen; history; text; internal format)

OFFSET	1,10
LINKS	Joseph Myers, Table of n, a(n) for n=1..70 Index entries for the Kaprekar map
CROSSREFS	Cf. A151949, A164725, A164726, A164731, A164732, A164733, A164734, A164736. Sequence in context: A174381 A184363 A331451 * A293933 A345057 A158976 Adjacent sequences: A164732 A164733 A164734 * A164736 A164737 A164738
KEYWORD	base,nonn
AUTHOR	Joseph Myers, Aug 23 2009
STATUS	approved

$$K\colon \mathbb{N} \to \mathbb{N},\ K(n) := \mathsf{reverse}(\mathsf{sort}(n)) - \mathsf{sort}(n)$$

$$\mathsf{K}\colon \mathbb{N} \to \mathbb{N},\ \mathsf{K}(n) := \mathsf{reverse}(\mathsf{sort}(n)) - \mathsf{sort}(n)$$

$$\mathsf{K}(64308654) = 86654430 - 03445668 = 83208762$$

$$\mathsf{K}\colon \mathbb{N} \to \mathbb{N},\ \mathsf{K}(n) := \mathsf{reverse}(\mathsf{sort}(n)) - \mathsf{sort}(n)$$

$$\mathrm{K}(64308654) = 86654430 - 03445668 = 83208762$$
$$\mathrm{K}(83208762) = 88763220 - 02236788 = 86526432$$

$$\mathsf{K}\colon \mathbb{N} \to \mathbb{N},\ \mathsf{K}(n) := \mathsf{reverse}(\mathsf{sort}(n)) - \mathsf{sort}(n)$$

$$\begin{aligned}
\mathsf{K}(64308654) &= 86654430 - 03445668 = 83208762 \\
\mathsf{K}(83208762) &= 88763220 - 02236788 = 86526432 \\
\mathsf{K}(86526432) &= 86654322 - 22345668 = 64308654
\end{aligned}$$

$$K\colon \mathbb{N} \to \mathbb{N},\ K(n) := \mathsf{reverse}(\mathsf{sort}(n)) - \mathsf{sort}(n)$$

$$\begin{aligned}
K(64308654) &= 86654430 - 03445668 = 83208762\\
K(83208762) &= 88763220 - 02236788 = 86526432\\
K(86526432) &= 86654322 - 22345668 = 64308654
\end{aligned}$$

$0, 0, 0, 0, 0, 0, 0, 1, 0, 4, 0, 10, 0, 20, 0, 36, 0, 60, 1, 94, 4, \ldots$

$$
\begin{aligned}
&n(n^7 + 39n^6 + 567n^5 + 226n^4 - 100848n^3 - 1284854n^2 - 6546176n - 12227835)a(n+1) + {} \\
&2(99n^7 + 4428n^6 + 72809n^5 + 500967n^4 + 501159n^3 - 11621783n^2 - 58155199n - 84302160)a(n+7) + {} \\
&(n-1)(6n^8 + 381n^7 + 9931n^6 + 137719n^5 + 1097530n^4 + 5002058n^3 + 11776711n^2 + 9284678n - 6080760)a(n) + {} \\
&(n^8 + 226n^7 + 9159n^6 + 144719n^5 + 929875n^4 + 132541n^3 - 27738695n^2 - 125383426n - 170659680)a(n+4) + {} \\
&2(18n^8 + 969n^7 + 20784n^6 + 225560n^5 + 1238373n^4 + 2176326n^3 - 10518941n^2 - 59504269n - 87910980)a(n+6) - {} \\
&2(18n^8 + 708n^7 + 8877n^6 + 27664n^5 - 165195n^4 + 189930n^3 + 18410426n^2 + 101779492n + 172019400)a(n+8) - {} \\
&(n^8 - 159n^7 - 8289n^6 - 145392n^5 - 1102782n^4 - 2287172n^3 + 16697390n^2 + 104082563n + 168604320)a(n+10) - {} \\
&(n^8 + 28n^7 + 303n^6 - 899n^5 - 72059n^4 - 869777n^3 - 4495129n^2 - 9073028n - 2055360)a(n+13) - {} \\
&(6n^9 + 316n^8 + 6371n^7 + 60052n^6 + 237203n^5 + 172850n^4 + 3432614n^3 + 57689364n^2 + 272160144n + 429549600)a(n+2) + {} \\
&(6n^9 + 393n^8 + 10431n^7 + 144408n^6 + 1113687n^5 + 4613145n^4 + 8034743n^3 - 3440238n^2 - 19869315n + 4527300)a(n+3) - {} \\
&(6n^9 + 334n^8 + 7167n^7 + 73093n^6 + 336551n^5 + 537411n^4 + 4538780n^3 + 66529054n^2 + 318939244n + 515211480)a(n+5) - {} \\
&(6n^9 + 339n^8 + 7612n^7 + 86220n^6 + 508691n^5 + 1427782n^4 + 2422001n^3 + 18545849n^2 + 103643100n + 181902720)a(n+9) + {} \\
&(6n^9 + 280n^8 + 4955n^7 + 42298n^6 + 181875n^5 + 503240n^4 + 3052754n^3 + 20868512n^2 + 68601160n + 85510800)a(n+11) - {} \\
&(6n^9 + 357n^8 + 8493n^7 + 102840n^6 + 662567n^5 + 2136399n^4 + 3682091n^3 + 17597644n^2 + 99139223n + 180349260)a(n+12) + {} \\
&(6n^9 + 298n^8 + 5751n^7 + 55339n^6 + 281223n^5 + 867801n^4 + 4158920n^3 + 29708202n^2 + 115380260n + 171172680)a(n+14) \stackrel{?}{=} 0
\end{aligned}
$$

$$a(n) \stackrel{?}{=} \frac{1}{40} \cdot \begin{cases} 3(243k^5 + 405k^4 + 35k^3 + 395k^2 - 318k + 40), & n = 18k, \\ k(729k^4 - 405k^3 - 615k^2 + 225k + 106), & n = 18k + 1, \\ 729k^5 + 1620k^4 + 735k^3 + 1320k^2 - 684k + 40, & n = 18k + 2, \\ k(729k^4 - 705k^2 + 136), & n = 18k + 3, \\ 3k(243k^4 + 675k^3 + 515k^2 + 565k - 118), & n = 18k + 4, \\ k(729k^4 + 405k^3 - 615k^2 - 225k + 106), & n = 18k + 5, \\ 3k(243k^4 + 810k^3 + 845k^2 + 790k + 32), & n = 18k + 6, \\ 3k(k+1)(243k^3 + 27k^2 - 142k + 12), & n = 18k + 7, \\ 729k^5 + 2835k^4 + 3705k^3 + 3405k^2 + 726k + 40, & n = 18k + 8, \\ 3k(k+1)(243k^3 + 162k^2 - 127k - 18), & n = 18k + 9, \\ 729k^5 + 3240k^4 + 5055k^3 + 4860k^2 + 1636k + 160, & n = 18k + 10, \\ 3k(k+1)(243k^3 + 297k^2 - 52k - 48), & n = 18k + 11, \\ 729k^5 + 3645k^4 + 6585k^3 + 6795k^2 + 2926k + 400, & n = 18k + 12, \\ 3k(k+1)(243k^3 + 432k^2 + 83k - 58), & n = 18k + 13, \\ 729k^5 + 4050k^4 + 8295k^3 + 9270k^2 + 4696k + 800, & n = 18k + 14, \\ 3k(k+1)(243k^3 + 567k^2 + 278k - 28), & n = 18k + 15, \\ 3(k+3)(243k^4 + 756k^3 + 1127k^2 + 734k + 160), & n = 18k + 16, \\ 3k(k+1)(243k^3 + 702k^2 + 533k + 62), & n = 18k + 17, \end{cases}$$

?

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A098926 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **a098926**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A098926 Permanent of the (0,1)-matrix with ij-th entry equal to zero iff (i=1,j=1),(i=1,j=2),(i=1,j=3),(i=2,j=3),(i=3,j=3),... In other words, the ij-th entry of the matrix is zero iff it is on the path which start from the entry (i=1,j=1) and moves in the matrix alternating 3 steps to the right to 3 steps down. +40 1

0, 2, 12, 90, 556, 5242, 42380, 479306, 4817484, 63779034, 767504524, 11661506218, 163541678156, 2806878055610, 44960579074956, 860568917787402, 15502269624912460, 327460573946510746, 6552868832109180044, 151436547414562736234, 3332986639447590230604, 83655126041771262574458 (list; graph; refs; listen; history; text; internal format)

OFFSET 3,2

LINKS Vaclav Kotesovec, Table of n, a(n) for n = 3..36

EXAMPLE a(3) = 12 because 12 is the permanent of the following 5 X 5 matrix:

```
[0 0 0 1 1]
[1 1 0 1 1]
[1 1 0 0 0]
[1 1 1 1 0]
[1 1 1 1 0]
```

PROG

```
(PARI) a(n)={my(M=matrix(n, n, i, j, j-i<>1 && (i%2==0 || abs(j-i-1)<>1)));
   matpermanent(M)} \\ Andrew Howroyd, Nov 05 2019
```

KEYWORD nonn

AUTHOR Simone Severini, Oct 19 2004

EXTENSIONS Terms a(11) and beyond from Andrew Howroyd, Nov 05 2019

STATUS approved

$$\begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

$$\mathrm{perm}(A) = \sum_{\pi \in S_n} \cancel{\mathrm{sgn}(\pi)} \prod_{i=1}^{n} a_{i,\pi(i)}$$

$$0, 2, 12, 90, 556, 5242, 42380, 479306, 4817484, 63779034, \ldots$$

$$(-13n^5 - 596n^4 - 10087n^3 - 79250n^2 - 292543n - 408476)a(n+1) + (-n^5 + 88n^4 + 3271n^3 + 38812n^2 + 192747n + 342724)a(n+3) + (17n^5 + 700n^4 + 11123n^3 + 84058n^2 + 297635n + 385900)a(n+5) + (-3n^5 - 128n^4 - 2099n^3 - 16452n^2 - 61615n - 88276)a(n+7) + (3n^5 + 125n^4 + 1993n^3 + 15181n^2 + 55475n + 78291)a(n+8) + (3n^7 + 161n^6 + 3539n^5 + 41281n^4 + 276187n^3 + 1060665n^2 + 2165448n + 1812816)a(n) + (-9n^7 - 483n^6 - 10670n^5 - 126072n^4 - 864460n^3 - 3458538n^2 - 7521075n - 6907077)a(n+2) + (9n^7 + 507n^6 + 11834n^5 + 148242n^4 + 1075988n^3 + 4523194n^2 + 10184921n + 9451433)a(n+4) + (-3n^7 - 185n^6 - 4802n^5 - 67928n^4 - 564684n^3 - 2754038n^2 - 7288033n - 8071719)a(n+6) \stackrel{?}{=} 0$$

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Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A253217 Number of n X n nonnegative integer arrays with upper left 0 and lower right its king-move distance away minus 2 and every value within 2 of its king move distance from the upper left and every value increasing by 0 or 1 with every step right, diagonally se or down. 1

0, 0, 1, 19, 268, 3568, 47698, 649712, 9023385, 127419681, 1823918697, 26398702645, 385582981615, 5674890516295, 84060883775765, 1252066289632643, 18738613233957420, 281620474177057788, 4248088188086420832 (list; graph; refs; listen; history; text; internal format)

OFFSET	1,4
COMMENTS	Diagonal of A253223.
LINKS	R. H. Hardin, Table of n, a(n) for n = 1..37
EXAMPLE	Some solutions for n=4:

```
..0..1..1..1....0..1..1..1....0..0..0..1....0..0..0..1....0..0..0..1
..0..1..1..1....0..1..1..1....0..0..1..1....0..0..0..1....0..0..1..1
..0..1..1..1....1..1..1..1....0..1..1..1....0..0..1..1....0..0..1..1
..1..1..1..1....1..1..1..1....1..1..1..1....1..1..1..1....1..1..1..1
```

CROSSREFS	Sequence in context: A036736 A016254 A016302 * A245237 A141942 A181043 Adjacent sequences: A253214 A253215 A253216 * A253218 A253219 A253220
KEYWORD	nonn
AUTHOR	R. H. Hardin, Dec 29 2014
STATUS	approved

Number of $n\times$ n nonnegative integer arrays with upper left 0 and lower right its king-move distance away minus 2 and every value within 2 of its king move distance from the upper left and every value increasing by 0 or 1 with every step right, diagonally SE or down.

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 1 & 2 & 2 & 3 & 4 & 5 & 5 \\ 2 & 2 & 2 & 2 & 3 & 4 & 5 & 5 \\ 2 & 2 & 3 & 3 & 3 & 4 & 5 & 5 \\ 3 & 3 & 3 & 3 & 3 & 4 & 5 & 5 \\ 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\ 4 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 1 & 2 & 2 & 3 & 4 & 5 & 5 \\ 2 & 2 & 2 & 2 & 3 & 4 & 5 & 5 \\ 2 & 2 & 3 & 3 & 3 & 4 & 5 & 5 \\ 3 & 3 & 3 & 3 & 3 & 4 & 5 & 5 \\ 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\ 4 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1\pm 2 & 2\pm 2 & 3\pm 2 & 4\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 1\pm 2 & 1\pm 2 & 2\pm 2 & 3\pm 2 & 4\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 2\pm 2 & 2\pm 2 & 2\pm 2 & 3\pm 2 & 4\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 3\pm 2 & 3\pm 2 & 3\pm 2 & 3\pm 2 & 4\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 4\pm 2 & 4\pm 2 & 4\pm 2 & 4\pm 2 & 4\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 5\pm 2 & 5\pm 2 & 5\pm 2 & 5\pm 2 & 5\pm 2 & 5\pm 2 & 6\pm 2 & 7\pm 2 \\ 6\pm 2 & 6\pm 2 & 6\pm 2 & 6\pm 2 & 6\pm 2 & 6\pm 2 & 6\pm 2 & 7\pm 2 \\ 7\pm 2 & 7\pm 2 & 7\pm 2 & 7\pm 2 & 7\pm 2 & 7\pm 2 & 7\pm 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 \pm 2 & 2 \pm 2 & 3 \pm 2 & 4 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
1 \pm 2 & 1 \pm 2 & 2 \pm 2 & 3 \pm 2 & 4 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
2 \pm 2 & 2 \pm 2 & 2 \pm 2 & 3 \pm 2 & 4 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
3 \pm 2 & 3 \pm 2 & 3 \pm 2 & 3 \pm 2 & 4 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
4 \pm 2 & 4 \pm 2 & 4 \pm 2 & 4 \pm 2 & 4 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
5 \pm 2 & 5 \pm 2 & 5 \pm 2 & 5 \pm 2 & 5 \pm 2 & 5 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
6 \pm 2 & 6 \pm 2 & 6 \pm 2 & 6 \pm 2 & 6 \pm 2 & 6 \pm 2 & 6 \pm 2 & 7 \pm 2 \\
7 \pm 2 & 7 \pm 2 & 7 \pm 2 & 7 \pm 2 & 7 \pm 2 & 7 \pm 2 & 7 \pm 2 & 5
\end{pmatrix}$$

$$
\begin{pmatrix} 0&0&0&1\\0&0&0&1\\0&0&0&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&0&0&1\\0&0&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&0&0&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&0&0&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&0&1&1\\0&0&1&1\\1&1&1&1 \end{pmatrix},
$$

$$
\begin{pmatrix} 0&0&0&1\\0&0&1&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&0&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&1&1&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\0&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&0&1\\1&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
$$

$$
\begin{pmatrix} 0&0&1&1\\0&0&1&1\\0&0&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&1&1\\0&0&1&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&1&1\\0&0&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&1&1\\0&1&1&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&0&1&1\\0&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
$$

$$
\begin{pmatrix} 0&0&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&1&1&1\\0&1&1&1\\0&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&1&1&1\\0&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix},
\begin{pmatrix} 0&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1 \end{pmatrix}.
$$

0, 0, 1, 19, 268, 3568, 47698, 649712, 9023385, 127419681, . . .

$$
\begin{gathered}
(201600n^9 + 4942080n^8 + 53078112n^7 + 327661728n^6 + \\
1280700480n^5 + 3285342016n^4 + 5528828352n^3 + 5883447104n^2 + \\
3591093120n + 957662208)a(n) + (-970200n^9 - 24199560n^8 - \\
264810744n^7 - 1667830872n^6 - 6659340648n^5 - 17470825688n^4 - \\
30096410912n^3 - 32804461872n^2 - 20514211488n - \\
5603970816)a(n+1) + (589050n^9 + 14827590n^8 + 163756656n^7 + \\
1040895564n^6 + 4194035058n^5 + 11101344742n^4 + 19289250308n^3 + \\
21198776056n^2 + 13360158000n + 3676219776)a(n+2) + (294525n^9 + \\
7319295n^8 + 79828578n^7 + 501335472n^6 + 1997003589n^5 + \\
5229549731n^4 + 8997110634n^3 + 9799013608n^2 + 6125859120n + \\
1673566848)a(n+3) + (-121275n^9 - 3053295n^8 - 33716268n^7 - \\
214212552n^6 - 862421763n^5 - 2280190003n^4 - 3956305720n^3 - \\
4340670060n^2 - 2730542400n - 749859264)a(n+4) + (6300n^9 + \\
163890n^8 + 1863666n^7 + 12150660n^6 + 50023284n^5 + 134779202n^4 + \\
237527338n^3 + 263895164n^2 + 167643648n + 46381248)a(n+5) \stackrel{?}{=} 0
\end{gathered}
$$

?