Gröbner Bases

Exercise Sheet 14 for January 28th, 2025

(1) Compute a strong Gröbner basis of the ideal generated by $\{2x, 3y\}$ over the integers.
Note: Mathematica might give a different result.

(2) Let $g_1 = 3y^2 - 2$, $g_2 = 2x - 1$, $g_3 = xy^2 + y^2 - 1$. Use the method from Case 2.2.2 of the proof of Theorem 13.10 to obtain, starting from the representation

$$-4x^2 + 6xy^2 - 2x = 2x^2 g_1 - 4xy^2 g_2 + 2x g_3$$

and from

$$s_{1,2} = x g_1 - y^2 g_2 = -g_2 + g_3,$$

a new representation with smaller complexity parameters. (Use the lexicographic order with $x > y$).

(3) Find a strong standard representation of

$$f = -4x^2 + 6xy^2 - 2x$$

by $G = \{3y^2 - 2, 2x - 1, xy^2 + y^2 - 1\}$.

We also discuss Problem 5 from Sheet 13.