

(* Sheet 2, Problem 4 *)



```
In[1]:= f = x^3 y^3 + 1;  
p = 1 + 3 x + 2 x^2 + x^2 y + x^3 y;  
q = x y^2 + x^2 y^2;
```

```
In[4]:= ShowP[f_] := MonomialList[f, {x, y}, IdentityMatrix[2]]
```

```
In[5]:= ShowP[f]
```

```
Out[5]= {x^3 y^3, 1}
```

```
In[6]:= ShowP[p]
```

```
Out[6]= {x^3 y, x^2 y, 2 x^2, 3 x, 1}
```

```
In[7]:= ShowP[q]
```

```
Out[7]= {x^2 y^2, x y^2}
```

```
In[8]:= PolynomialReduce[f, {p, q}, {x, y}, MonomialOrder -> IdentityMatrix[2]]
```

```
Out[8]= {{y^2, -2 - y}, {1 - y^2 - x y^2 + x y^3}}
```

```
In[9]:=
```

(* Sheet 2, Problem 5 *)

```
In[10]:= f = x^3 y^2;  
p = 1 + x^3 y + 3 x^2 y^5;  
q = 2 x^2 y + x^2 y^2;
```

```
In[13]:= ShowP[f_] := MonomialList[f, {x, y}, IdentityMatrix[2]]
```

```
In[14]:= ShowP[f]
```

```
Out[14]= {x^3 y^2}
```

```
In[15]:= ShowP[p]
```

```
Out[15]= {x^3 y, 3 x^2 y^5, 1}
```

```
In[16]:= ShowP[q]
```

```
Out[16]= {x^2 y^2, 2 x^2 y}
```

```
In[17]:= PolynomialReduce[f, {p, q}, {x, y}, MonomialOrder -> IdentityMatrix[2]]
```

```
Out[17]= {{y, -48 + 24 y - 12 y^2 + 6 y^3 - 3 y^4}, {-y + 96 x^2 y}}
```

```
In[18]:= (* Sheet 2, Problem 6 *)
```

```
In[19]:= f = x^2 y + x y^2 + y^2;  
p = x y - 1;  
q = y^2 - 1;
```

```
In[22]:= ShowP[f_] := MonomialList[f, {x, y}, IdentityMatrix[2]]
```

```
In[23]:= ShowP[f]
```

```
Out[23]= {x^2 y, x y^2, y^2}
```

```
In[24]:= ShowP[p]
```

```
Out[24]= {x y, -1}
```

In[25]:= **ShowP [q]**

Out[25]= $\{y^2, -1\}$

In[26]:= **PolynomialReduce [f, {p, q}, {x, y}, MonomialOrder $\rightarrow$ IdentityMatrix[2]]**

Out[26]= $\{\{x+y, 1\}, 1+x+y\}$

In[27]:= **Expand [f - x q - q - x p]**

Out[27]= $1 + 2x$

In[28]:= **Expand [(x+1) q + x p + 1 + 2 x]**

Out[28]= $x^2 y + y^2 + x y^2$

In[29]:= **GroebnerBasis[{p, q}, {x, y}, MonomialOrder $\rightarrow$ IdentityMatrix[2]]**

Out[29]= $\{-1+y^2, x-y\}$

In[30]:= **(* Example for Lecture on Oct 28, 2020 *)**

In[31]:= **f**

Out[31]= $x^2 y + y^2 + x y^2$

In[32]:= **ShowP [f]**

Out[32]= $\{x^2 y, x y^2, y^2\}$

In[33]:= **ShowP [p]**

Out[33]= $\{x y, -1\}$

ShowP [q]

$\{y^2, -1\}$

In[34]:= **ShowP [f - x p]**

Out[34]= $\{x y^2, x, y^2\}$

In[35]:= **ShowP [f - x p - y p - 1 q]**

Out[35]= $\{x, y, 1\}$