

**Einführung in die Algebra**  
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**Übungsaufgaben 6, June 17th**

1)

Build the operation tables for  $GF(8)$  as shown in 18.18 on page 54 in the script.

2)

Find a generator of the multiplicative group in  $GF(8)$  and verify if you actually can build  $GF(8)$  with this generator.

3)

*Definition:* Let  $L$  be an extension field of  $K$ . The field  $L$  can be interpreted as a vector space over  $K$ . We denote  $\dim_K L$  as  $[L:K]$  and call it the “degree of the extension field.”

Find the degrees and a basis for:

(a)  $[C : \mathbb{R}]$                       (b)  $[Q : Q(\sqrt{2})]$  where  $Q(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in Q\}$

4)

(a) Is  $K = \{a + b\sqrt[3]{2} \mid a, b \in Q\}$  an extension field of  $Q \leq \mathbb{R}$ ? If so, determine  $[K : Q]$ .

(b) Like (a) for  $K = \{a + b\sqrt[3]{2} + c\sqrt[3]{4} \mid a, b, c \in Q\}$ .

5)

Determine the code defined by  $\begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$  (i.e. list all the code words).

6)

Let  $P := \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$  be the control matrix of a code.

- (a) Determine a basis of the corresponding code  $C$ .
- (b) How many test symbols are there in a code word?
- (c) Determine all code words.
- (d) How many errors can be detected / corrected?
- (e) Determine the information and correction rate.

7)

Let the control matrix  $H := \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$  define a code. Decode  $y = 1101011$ .

8)

Think of a code that encodes words by simply repeating each word, i.e. "sending it twice", for example 101 is encoded to 101101. Is that a good code?

(I.e. determine information and correction rate for general word length  $n$ ).