Discrete Mathematics

368.115

Exercise sheet 4 for October 28, 2016

Solve 3 out of 5!

(1) Design a deterministic finite automaton that recognizes the language L_1 that contains exactly those $x \in \{a, b, c\}^*$ that contain all three letters a, b, c, and have the additional property that the first occurrence of a comes before the first occurrence of b, and the first occurrence of b comes before the first occurrence of c.

(2) Design a deterministic finite automaton that recognizes the complement of L_1.

(3) We say that $x_1 x_2 \ldots x_m$ is a *scattered subword* of $y_1 y_2 \ldots y_n$ if there are $w_0, \ldots, w_m \in A^*$ with $y_1 y_2 \ldots y_n = w_0 x_1 w_1 x_2 w_2 \ldots x_m w_m$. Show that the set of all words in $\{a, b\}^*$ that do not have aba as a scattered subword is a recognizable language.

(4) On a deterministic finite automaton $M = (A, Q, I, J, R)$ over A, define two states q_1, q_2 to be *equivalent* ($q_1 \sim q_2$) if for all $x \in A^*$, we have

$$\delta(x, q_1) \in J \Longleftrightarrow \delta(x, q_2) \in J.$$

(a) Show that for all $a \in A$, we have $q_1 \sim q_2 \Rightarrow \delta(a, q_1) \sim \delta(a, q_2)$.

(b) Give an example of a finite automaton such that for all $a \in A$, $\delta(a, q_1) \sim \delta(a, q_2)$, and $q_1 \not\sim q_2$.

(c) Show that a final state can never be equivalent to a nonfinal state.

(d) Give an example of a deterministic finite automaton with two nonequivalent final states.

(5) On a deterministic finite automaton $M = (A, Q, I, J, R)$ over A, define two states q_1, q_2 to be equivalent ($q_1 \sim q_2$) if for all $x \in A^*$, we have $\delta(x, q_1) \in J \Leftrightarrow \delta(x, q_2) \in J$. Since A^* is infinite, this definition does not immediately yield a procedure to decide whether two given q_1, q_2 are equivalent. Design an algorithm that decides whether $q_1 \sim q_2$. (The focus is not on the efficiency of this algorithm, but on its existence).