Discrete Mathematics

368.115

Exercise sheet 3 for October 21, 2016

Solve 3 out of the following 5 problems!

(1) (a) Design a nondeterministic finite automaton M_1 that recognizes the language $L_1 = \{x \in \{a, b\}^* \mid x \text{ has } aba \text{ as a subword}\}$.

(b) Use the subset construction to transform M_1 into a deterministic finite automaton D_1 that recognizes L_1

(2) (a) Design a nondeterministic finite automaton M_2 that recognizes the language $L_2 \subseteq \{a\}^*$ given by

$$L_2 = \{a^n \mid n \in \mathbb{N}_0 \text{ and } (n \equiv 1 \pmod 2 \text{ or } n \equiv 0 \pmod 3)\}.$$

(b) Use the subset construction to transform M_2 into a deterministic finite automaton D_2 that recognizes L_2.

(3) Let $M = (A, Q, I, J, R)$ be a deterministic finite automaton that recognizes the language $L \subseteq A^*$, and let $q_0 \in Q$ be the unique element of I; hence $M = (A, Q, \{q_0\}, J, R)$. For each $q \in Q$, let M_q be the DFA $(A, Q, \{q\}, J, R)$ (hence $M_{q_0} = M$). Let $L_q = \mathrm{Lang}(M_q)$. Show that for all $x \in A^*$, we have $x^{-1}L = L_{\delta(x,q_0)}$.

(4) [1, p. 63] ("Pumping Lemma" for recognizable languages.) Let L be recognizable. Show that there exists a natural number n such that every word $w \in L$ with length at least n can be written as $w = xyz$, with the length of y at least 1, such that for all natural numbers k, $xy^kz \in L$ (where y^k denotes the k-fold concatenation of y with itself).

Use the pumping lemma to show that $\{a^{n^2} \mid n \in \mathbb{N}\}$ is not a recognizable language.

(5) For a DFA M that recognizes a language L, construct a NFA M' that recognizes $\{a^n \mid n \in \mathbb{N} \text{ and } L \text{ contains at least one word of length } n\}$.

References

[1] Nicholas Pippenger. *Theories of computability.* Cambridge University Press, Cambridge, 1997.