Discrete Mathematics
368.115
Exercise sheet 2 for October 18, 2016

Solve 5 out of the following 7 problems![1]

(1) (a) Design a deterministic finite automaton that recognizes $L_1 = \{x \in \{a,b\}^* : x \text{ contains } aa \text{ or } x \text{ contains } bb \text{ as a subword}\}$. Hence $L_1 = \{aa, bb, aaa, aab, abb, baa, bba, bbb, \ldots\}$.

(b) Design a deterministic finite automaton that recognizes the language L_2 of all words over $\{a,b\}$ such that two consecutive letters are never the same. Hence

$$L_2 = \{\varepsilon, a, b, ab, ba, aba, bab, abab, baba, ababa, babab, \ldots\}.$$

(2) For the language $L_2 = \{\varepsilon, a, b, ab, ba, aba, bab, abab, baba, ababa, babab, \ldots\}$ given above compute the sets $x^{-1}L_2$ for all $x \in \{a,b\}^*$. *Hint:* There are 4 different sets.

(3) Let L_3 be the language $\{x \in A^* \mid x \text{ has the same number of } a \text{ and } b\text{'s}\}$. Hence $L_3 = \{\varepsilon, aa, bb, aabb, abab, abba, baab, baba, bbaa, \ldots\}$. For each $n \in \mathbb{N}$, compute $(a^n)^{-1}L_3$. Conclude that L_3 is not a finite state language.

(4) For each of the languages L_1, L_2, and L_3 and for each $n \in \mathbb{N}$, give a formula for the number of words of length n in the language.

(5) (cf. [1]) Design a finite automaton that recognizes $\{x_1x_2\ldots x_n \in \{0,1\}^* : 3 \mid \sum_{i=1}^{n} x_i \cdot 2^{n-i}\}$. Hence the automaton should accept exactly the binary representations of multiples of 3.

(6) Given a deterministic finite automaton M that recognizes L, how can you determine whether

(a) $L = \emptyset$,

(b) $L = A^*$,

(c) $\varepsilon \in L$,

(d) L contains a word of length at most 5,

(e) L is finite. *Hint:* Call a state *quick* if a final state can be reached from it.

(7) Prove the following fact: For every $n \in \mathbb{N}$ there is $N \in \mathbb{N}$ such that for all semigroups S with at most n elements and for all $x \in S$, x^N is an idempotent element of S.

References

[1] D. C. Kozen. *Automata and computability.* Undergraduate Texts in Computer Science. Springer-Verlag, New York, 1997.

[1] This means: 5 = 100%; Grade 4 requires 50% in total, over the whole term.