Discrete Mathematics
368.115
Exercise sheet 11 for January 13, 2017

Solve 3 out of the 5 new problems!

(1) [1, Exercise 5.8 on page 100].
(2) [1, Exercise 5.12 on page 100].
(3) [1, Exercise 6.22 on page 129].
(4) Show that every tournament $T = (V, \rho)$ with $|V| \geq 3$ has a vertex v such that for all $x \in T \setminus \{v\}$, there exists $y \in T \setminus \{v, x\}$ such that $(v, x) \in \rho$ or $((v, y) \in \rho$ and $(y, x) \in \rho)$.
(5) In the lecture, we proved that a graph $G = (V, E)$ with $n := |V| \geq 3$ is Hamiltonian if $\delta(u) + \delta(v) \geq n$ for all $u, v \in V$ with $u \neq v$. Give a direct proof that such a graph must be connected.

And we still want to solve:

> Let (V, E) be a connected graph, and let $c : E \to \mathbb{R}^+$ be the cost function for the edges. We assume that all edge costs are distinct (i.e., c is injective). Prove that (V, E) contains exactly one minimal spanning tree.

References

[1] D. Mašulović. The discrete charm of discrete mathematics. Lecture notes for a course at JKU Linz, Austria, 2006; available at `http://www.algebra.uni-linz.ac.at/Students/DiskreteMathematik/ws16/`, 2006.