Discrete Mathematics
368.115
Exercise sheet 1 for October 7, 2016

1. Design a deterministic finite automaton that recognizes $L = \{x \in \{a, b\}^* :$ the symbol before the last symbol is a $b\}$.

2. Design a nondeterministic finite automaton that recognizes $L = \{x \in \{a, b\}^* :$ the symbol before the last symbol is a $b\}$, possibly with at most 3 states.

3. (cf. [1, p. 301]) Design a deterministic finite automaton that recognizes the set of strings over the alphabet $\{a, b\}$ containing at least two occurrences of two consecutive b's, overlapping permitted (e.g., the string bbb should be accepted).

4. Let $L \subseteq A^*$, and let $u \in A^*$. We define

$$u^{-1}L := \{x \in A^* : ux \in L\}.$$

   Let $a, b \in A$, $u \in A^*$. Show:

   (a) $b^{-1}(a^{-1}L) = (ab)^{-1}L$,

   (b) $u^{-1}(L_1 \cup L_2) = (u^{-1}L_1) \cup (u^{-1}L_2)$,

   (c) $A^* \setminus (u^{-1}L) = u^{-1}(A^* \setminus L)$.

5. Let L be a language that is recognized by a deterministic finite automaton with q states. Suppose that L contains no word with less then q letters. Show that L is empty.

References

[1] D. C. Kozen. *Automata and computability*. Undergraduate Texts in Computer Science. Springer-Verlag, New York, 1997.