ALGEBRA IN AGRICULTURE

THE PROBLEM:

We have 7 ingredients for a fertilizer:

a, b, c, ... , g

Which combination of these ingredients should be taken in order to get a maximal yield?

SILLY SOLUTION:

We test on small fields (not in the algebraic sense!!):

1 field without any ingredient

7 fields with precisely 1 ingredient

21 fields with 2 ingredients

……………………………………

1 field with all 7 ingredients

Pretty silly: We need 1+7+21+ … +1 = 128 experimental fields.

Much too costly !!!

MUCH BETTER SOLUTION:

We only test a few selected combinations of the ingredients, observing two rules of fairness:

* Each ingredient should get the same chance to „show its power“

* Each experimental field should get the same number of ingredients

Sounds good, but how can we get that ???
We need a BIB-design!

We compute in a strange way ...

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

.	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	1	4	4	2
2	0	2	4	2	1	1	4
3	0	3	6	3	5	5	6
4	0	4	1	4	2	2	1
5	0	5	3	5	6	6	3
6	0	6	5	6	3	3	5

„Blocks“ arise:

block 1: 1,2,4

block 2: 2,3,5

block 3: 3,4,6

block 4: 4,5,0

....

block 7: 0,1,3

block 8: 3,5,6

block 9: 4,6,0

block 10: 5,0,1

block 11: 6,1,2

....

block 14: 2,4,5

These blocks lead to the incidence matrix:

block 1: 1,2,4

block 2: 2,3,5

block 3: 3,4,6

block 4: 4,5,0

……………..

block 7: 0,1,3

block 8: 3,5,6

block 9: 4,6,0

block 10: 5,0,1

block 11: 6,1,2

………………

block 14: 2,4,5

$$\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}$$

We get a balanced incomplete block design (BIB-Design):

- **We have a set P of v „points“ (here, v=7)**
- **We have a collection B of b „blocks“ (subsets of P) (here, b = 14)**

Such that

- **Each set in B has the same cardinality k (here, k=3)**
- **Each point is in the same number r of blocks (here, r=6)**
- **Each pair of different points is in the same number of blocks (here, λ =2)**

This connection between planar near-rings and BIB-Designs is due to

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Giovanni FERRERO

Wenfong KE

Gerhard WENDT

Back to the „roots“: the design of the experiment:

$$\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a				x		x	x		x	x		x		
b	x				x		x			x	x		x	
c	x	x				x					x	x		x
d		x	x				x	x				x	x	
e	x		x	x					x				x	x
f		x		x	x			x		x				x
g			x		x	x		x	x		x			

Experimental design with yields:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a				x		x	x		x	x		x		
b	x				x		x			x	x		x	
c	x	x				x					x	x		x
d		x	x				x	x				x	x	
e	x		x	x					x				x	x
f		x		x	x			x		x				x
g			x		x	x		x	x		x			
	12.3	**14.1**	**12.1**	**14.9**	**11.1**	**13.6**	**12.5**	**11.2**	**13.9**	**13.5**	**11.3**	**13.9**	**12.2**	**15.1**

Record: **15.1**   Reference field: yield = 10.5

This gives the contributions of the ingredients (95%-level):

PValue = 0.000862841; { Estimate, CI} =

- {1, 10.50, {9.32 ,11.68}},
- {a, 1.93, {1.25 ,2.61}},
- {b, -0.42, {-1.10 ,0.26}},
- {c, 1.43, {0.75 ,2.11}},
- {d, 0.36, {-0.32 ,1.04}},
- {e, 1.48, {0.80 ,2.16}},
- {f, 1.33, {0.65 ,2.01}},
- {g, -0.34, {-1.02 ,0.34}}

Taking only a, c, e, and f gives the yield = 16.67

Are the ingredients independent?

NO ! c and f have a significant synergy effect:

PValue = 2.74503×10^{-7} { Estimate, CI} =

- {1, 10.56, {10.20, 10.92}},
- {a, 2.24, {1.92, 2.57}},
- {c, 0.77, {0.36, 1.18}},
- {e, 1.31, {0.98, 1.63}},
- {f, 0.67, {0.21, 1.08}},
- {c*f, 1.95, {1.26, 2.65}}

Taking a, c, e, and f gives the true yield = 17.50

Parameters of some designs:

Let N be planar of size v, and, for a in N, ***a*** be the map from N to N, sending n to na.

Then the set of all non-zero maps ***a*** forms a fixed-point-free group of size k (say) w.r.t. composition. We get a Frobenius group and hence a „Ferrero pair“.

From that we obtain various BIB-designs, e.g., one with parameters (v, v(v-1)/k, v-1, k, k-1).

Special case:

Let F be a finite field of size q, and let q-1 = s*t

By changing the multiplication in F suitably, we obtain various BIB-designs with parameters

(q, qs/(t+1), s, t+1, 1) or

(q, qs , q+s-1, t+1, t+1) or

(q, qs , q-1 , t, t-1) or ……

„Finite Fields“

Another „Ferrero Pair“

Farmers go Algebra

We look again at the incidence matrix:

$$
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

A B C D E F G H I J K L M N

ALG = 0110100 1011000 1101000

Also: „column codes“ instead of „row codes“:

$$\begin{array}{cc}
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
&
\begin{matrix}
\mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \\ \mathbf{G}
\end{matrix}
\end{array}$$

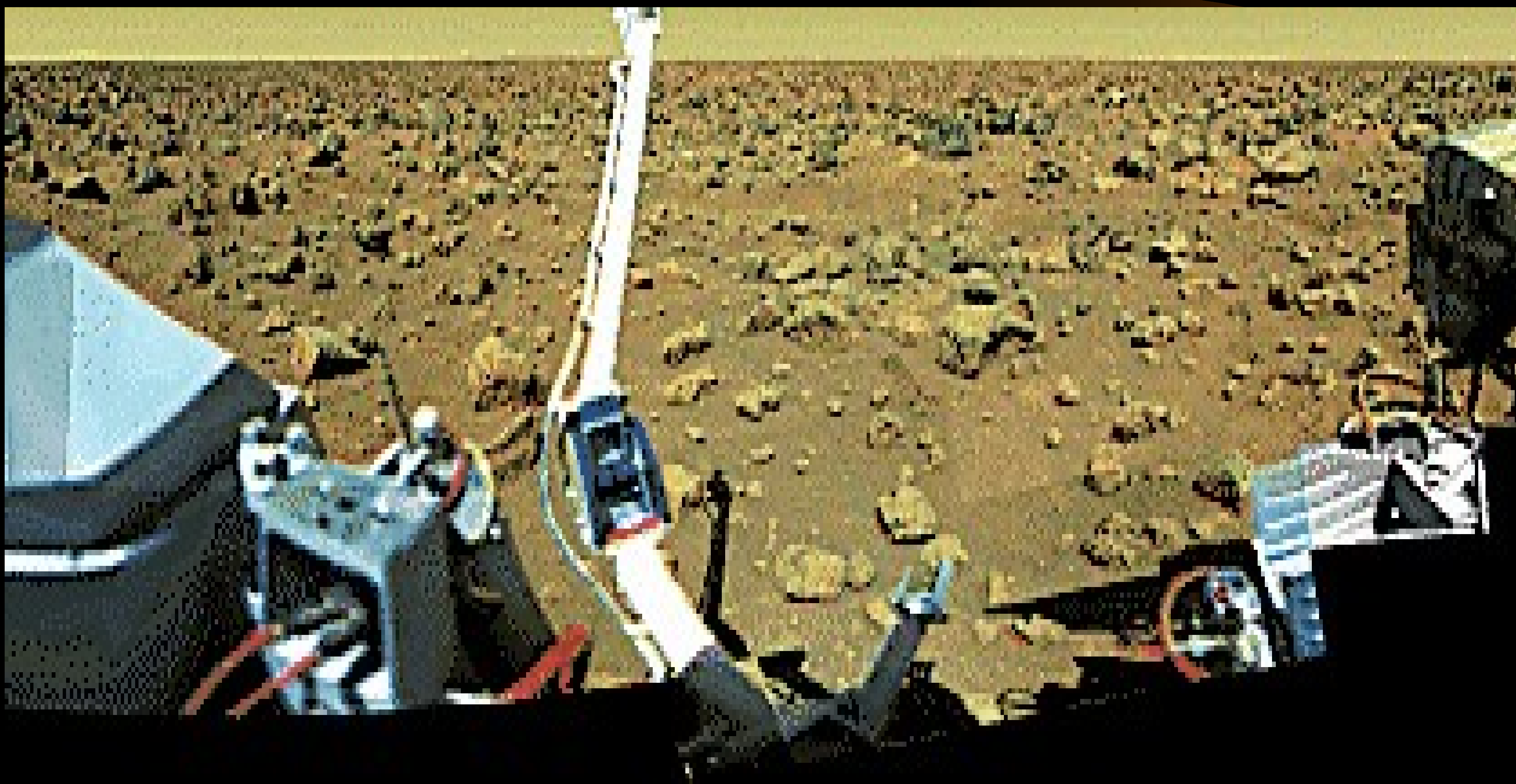
Maximal codes:

A code of length n, minimal distance d, and constant weight w is ***maximal***, if it has A(n,d,w) codewords.

Result: All row codes from incidence matrices of BIB-designs are maximal.
The same is true for all column codes from BIB-designs with $\lambda=1$.

So, e.g.,
$A(qs, 2(q+s-t-2), q+s-1) = q$

This gives „optimal“ error-correcting codes, useful in many situations, e.g., image transmission from Mars:



In all the years, only one error could not be corrected.

A dreadful error …